Volume Functions of Historical Texts and the Amplitude Correlation Principle

Anatoliy T. Fomenko*

Department of Mathematics and Mechanics, Moscow University, Leninskie Gory Moscow, 119889, U.S.S.R.

and

Svetlozar T. Rachev**

***Statistics &Applied Probability Program, University of California, Santa Barbara, California 93106, U.S.A.*

Abstract: A new empirico-statistical model is suggested to distinguish dependent narrative texts from independent narrative texts by means of their volume functions. A "regard for information" principle and an "amplitude correlation" principle are formulated. The model and both principles are examined experimentally using specific historical texts.

Key Words: historical texts, empirico-statistical methods, Weibull-Gnedenko distribution.

The present paper develops further the ideas of "statistical chronology" of narrative texts, which include *inter alia* historical texts (chronicles, annals, etc.). A foundation for this new branch of empirical "limited-sample" statistics was laid in a series of studies by Fomenko (1980, 1981a, 1981b, 1983). These new methods were evolved by Fedorov and Fomenko in (1986), by Kalashnikov, Rachev and Fomenko (1986) and successfully applied by Morozova (1985) to specific problems associated with analyzing source documentation and literature. Methods developed along these lines can also be adapted to the studies of long codes, biological problems of recognizing "like" patterns in gene chains, etc. One major problem addressed by the new empirico-statistical methodologies is the recognition of dependent and independent texts.

Dependent and Independent Texts, Text Volume Functions

According to Fomenko (1980) we shall call two historical texts X and Y "dependent" if they describe essentially the same events on the same time interval [A, B] of the history of a region.

We shall call two texts X and Y "independent" if they describe events on significantly different intervals [A, B] and [C, D], or events that took place in different regions. In general, for simplicity we shall assume that we are dealing with texts that describe intervals of equal length, i.e., we let $B - A = D - C$.

Consider a text X describing events on an

Anatoliy T. Fomenko (Dr. Sci. in mathematics, Lomonosov State University) is a Professor at Lomonosov State University, Moscow. He is author of twelve monographs and textbooks, and is co-author of the two-volume Modern Geometry *(Springer-Verlag). He is also artist and historian.*

Svetlozar T. Rachev (Dr. Sci., Steklov Mathematical Institute) is a Professor at the University of California, Santa Barbara. He has published more than seventy papers and is co-author of the monograph Mathematical Models for Construction of Queuing Models *(Moscow, 1988). His main interests are: stability of stochastic models, theory of probability metrics, queuing theory and survival models.*

Computers and the Humanities **24**: 187—206, 1990.
© 1990 *Kluwer Academic Publishers. Printed in the Netherlands.*

interval [A, B]. Let t be a year ranging between A and B. Let X(t) be the part of the text that relates to the events of the year t. For many texts (particularly those with which we shall be dealing) the partitioning into fragments X(t) is specified by the chronicler himself. For brevity, we shall call fragments X(t) *chapters*. We may calculate the volume of each chapter in some units, say in lines. Therefore, given a text X partitioned into chapters we can compute the volumes of these chapters, vol X(t). We shall call the resulting function vol X(t), *volume function for historical text.*

The Principle of Correlation of Maxima

The maxima correlation principle was formulated as follows (see Fomenko 1980, 1981, 1985a):

(a) If two texts X and Y are dependent, then their volume functions exhibit "splashes" that are essentially simultaneous, i.e. the local maxima of vol X(t) and vol Y(t) correlate.

(b) If two texts X and Y are independent then the local maxima of their volume function do not correlate.

With this principle only the locations of the volume function maxima (splashes) were taken into consideration, while their amplitudes were ignored. In other words, only "highly directional data" (i.e. splash points) were analyzed. Kalashnikov, Rachev and Fomenko (1986) extended the general idea of volume function correlation for dependent texts to the volume functions themselves, i.e. their amplitudes were taken into account. Because this study involved a significantly larger volume of data than the maxima correlation principle, the verification of their hypothesis called for a computational experiment based on specific texts. The algorithms for volume function comparison as proposed by Kalashnikov, Rachev and Fomenko (1986) proved to be quite efficient when the texts compared had approximately equal volumes. For texts of considerably different volumes, however, the pattern was sometimes blurred. In this paper we extend these ideas to infer intrinsic laws governing the behavior of volume functions for sufficiently large narrative texts.

Monotone Information Loss Principle

The maxima correlation principle is derived from the fact that different chroniclers, in describing the same historical period, use mainly the same stock of information (texts) which has survived. For this reason they describe in greater detail those years for which more texts have survived and *vice versa.* In other words, in the previous works the analysis focused only on the splash itself, i.e. on the fact that a chronicler gathered for a given year especially detailed information. This results in virtually simultaneous splashes in the volume functions for texts written by different chroniclers. On the other hand, it is obvious that the splash magnitude (i.e. the volume function amplitude) may vary considerably since emphasis on the events of a given year may vary from one chronicler to another. As a result, when two volume graphs are compared the splashes may occur simultaneously, while their amplitudes may differ significantly and the splashes themselves may appear quite dissimilar. Here, we aim to show that there are, nevertheless, intrinsic volume function "invariants" that are "similar" for dependent texts and "dissimilar" for independent texts. These invariants are different in nature from the local maxima distribution on the interval [A, B].

We will now present some general considerations which will form the basis for the new method of volume function comparison. We shall briefly discuss possible models of mechanisms which influence chroniclers writing long texts (and of which they would not be aware).

Let C(t) be the total volume of texts relevant to the year t and written by contemporaries. The graph C(t) (Figure 1) provides an illustration for the distribution of the volume of information created during the historical period of interest. Of course, we cannot know the exact form of the graph C(t) since texts are lost or destroyed with the passage of time. This is why only part of the total amount of information has survived. Let X denote a chronicler who is not contemporary with the epoch [A, B] but is willing to "historify" it. Let $T = T(X)$ be the year in which he starts to write a chronicle of [A, B]. We may assume $T \geqslant B$. Let $C_T(t)$ be the volume of texts that survives from epoch t within the internal [A, B] until time T. In other words, $C_T(t)$ is the surviving texts at time T. Since the major mechanism governing the evolution of the primary stock is the loss of texts, we can

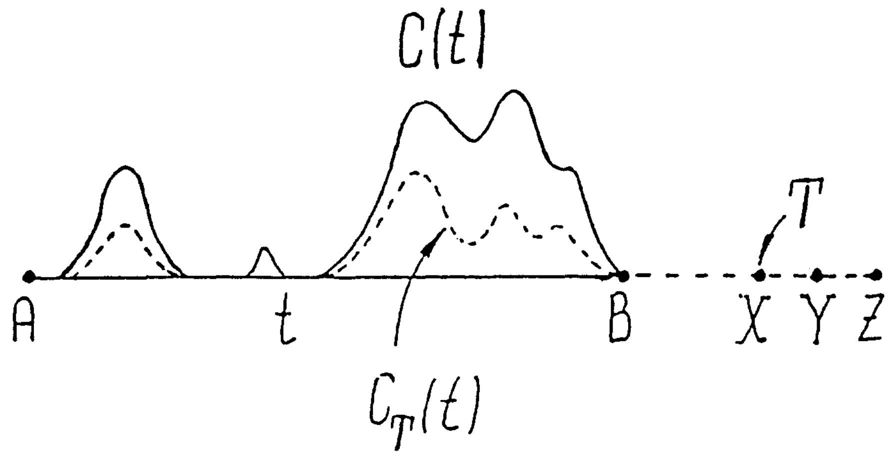

Figure 1.

assume that $C_T(t) \leqslant C(t)$, i.e. the primary stock C(t) for the most part can only decrease with time T. The farther we move in time from the epoch [A, B] of interest, the fewer documents from this epoch usually remain and the less we can learn about it. Here the *monotone information loss principle* is in force (for details see Fomenko, 1980, 1981a, 1981b, 1985). The values of $C_T(t)$ constitute a graph (a dashed line below the graph C(t) in Figure 1) which is *called the surviving information stock graph* (describing the events of epoch [A, B]).

Note that the idiosyncracies of a chronicler have little effect on the distribution of local maxima since these points are close to the splash points on $C_T(t)$ and, consequently, to the splash points on the primary graph C(t). This illustrates why splash points on the graphs due to different chroniclers relating the events of the same epoch are close to each other (Figure 2). Another mechanism which actually stems from the general principle of gradual loss of textual informations was considered in the basic monograph by Morozov (1924) and was formulated in modern terms by Fomenko (1981a, 1983) as the *frequency decay principle.* Since now we are interested in the *amplitudes* of volume functions, we shall try to find a new, more basic law governing the distribution of such amplitudes and which are essentially idiosyncracies of any particular chronicler.

Rich and Poor Texts, Rich and Poor Text Zones

The investigation of volume graph amplitudes in Kalashnikov, Rachev and Fomenko (1986) has shown that comparing rich texts with poor ones requires a great deal of care and does not always yield unambiguous results. Therefore, if we are to interpret the "epoch invariants" as we identify them, we have to separate these two categories of text. A qualitative definition of rich and poor texts is intuitively clear (Figure 3). A text in which most of volumes vol X(t) are nil, i.e. the majority of years are not described, will be called *poor.* Conversely, a text in which most of volumes vol X(t) are distinct from zero and quite large, i.e. the chronicler conveys much information on epoch [A, B], will be called *rich.* However, in the sequel we shall be dealing with actual historical texts describing long time periods (consisting of tens or hundreds of years). Such texts are sometimes too variable to classify unambiguously either as rich or poor. Therefore, we must introduce the notion of *rich and poor zones* of a text. Figure 4 exemplifies a text volume graph in which the beginning is clearly poor while the rest of the graph is rich. A part of the text in which most of the volumes vol X(t) are zero, i.e. most of the years in that historical period have not been described by the chronicler, would be called a *poor zone.* Equivalently, a part of the text in which most of volumes are sufficiently distinct from zero would be called a *rich zone.* Experience gained from investigating specific chronicles shows that the following observation is typical: the initial part of a long text is a poor zone, while the final part is a rich zone (see Figure 4). Such situations may occur because the

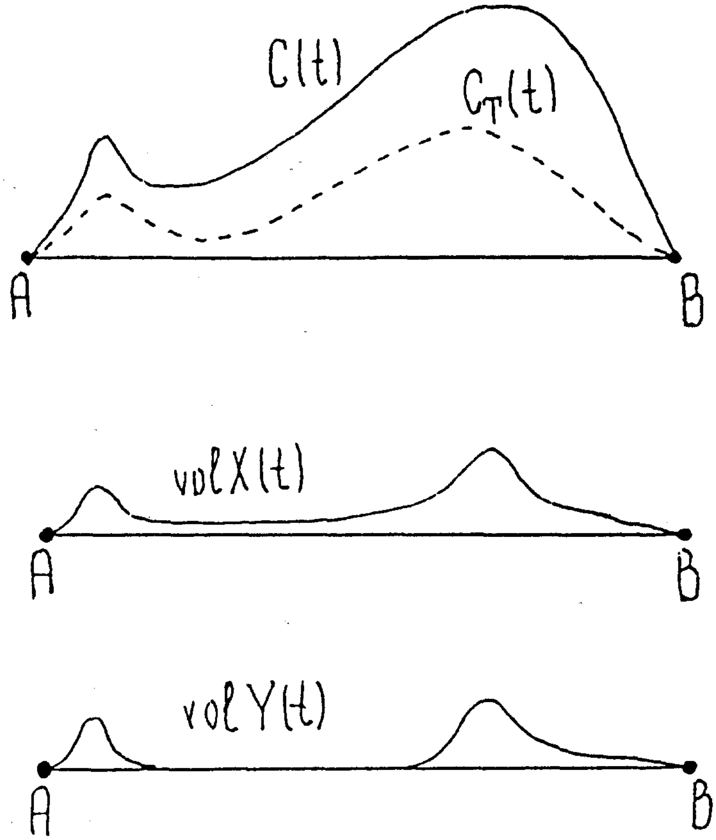

Figure 2.

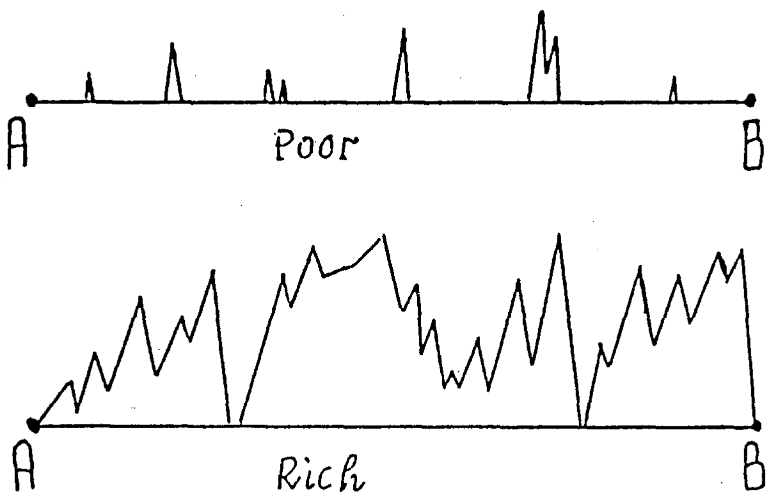

Figure 3.

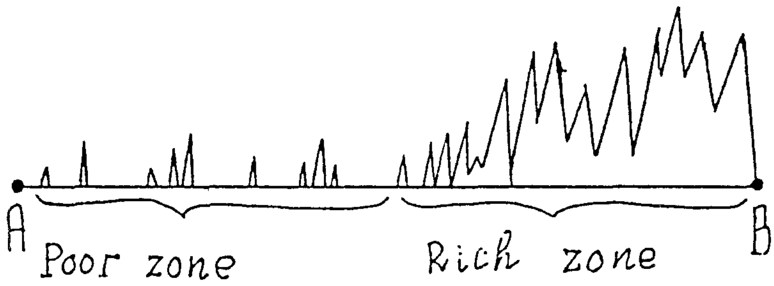

Figure 4.

farther we delve into the past the less is known about it. This is one reason why a chronicler often starts by writing about the periods of which he knows "something" even though the data available to him are scarce. As a result the amplitude of such a volume graph tends to increase gradually (for left-to-right movement) and a rich zone soon follows. This pattern is not always followed, however, since some chronicles have a poor zone in the middle region of the text (see Figure 5).

Consider an arbitrary volume graph for a chronicle X on [A, B]. For a year t let the volume vol X(t) be zero. If this zero lies to the right of the first nonzero value of the volume graph (e.g., β in Figure 6), then we shall call it *significant.* Conversely, if the zero t lies to the left of the first nonzero value of the volume graph (e.g. α in Figure 6), it will be termed *insignificant.* In the sequel, when investigating a particular chronicle, we shall take as the left point A (on the time axis) a year for which vol X(A) $\neq$ 0 (i.e. the year actually mentioned on the text X).

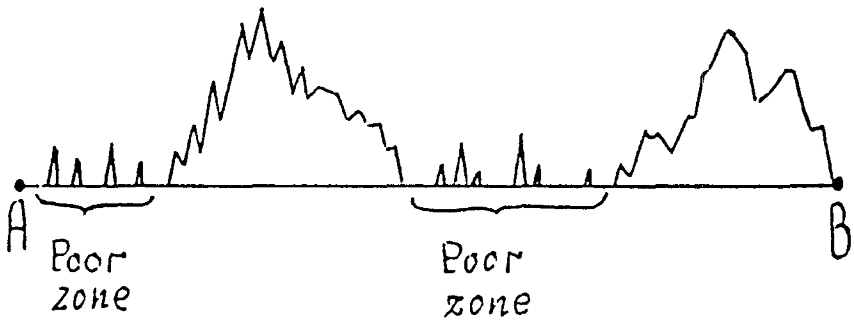

Figure 5.

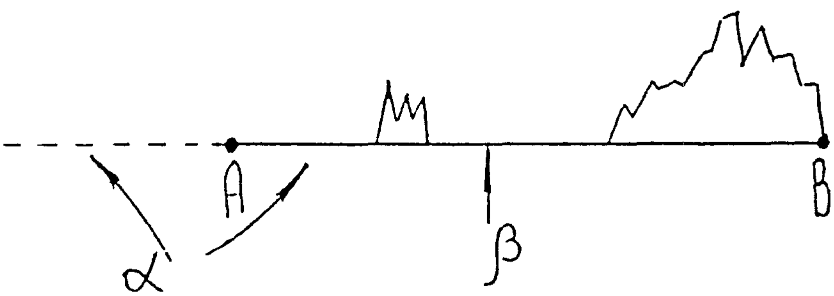

Figure 6.

"Regard-for-Information" Principle and Principle of Amplitude Correlation

Consider a fixed time interval [A, B] and a chronicler X living in the year $T(X) \geqslant B$ and describing the events of epoch [A, B] (Figure 7). To do so he first examines the surviving stock $C_T(t)$ of information on epoch [A, B]. This stock consists of rich and poor zones (Figure 7). Using the surving stock the chronicler covers both rich and poor zones. At this point we advance the main idea which forms the basis for the rest of our investigation: *the chronicler treats the rich and the poor zones differently.* First we shall give a brief formulation of this new hypothesis and then discuss it in detail.

The regard-for-information principle: "the regard for information varies inversely with its volume."

The explanation is as follows: when describing poor zone events the chronicler, of necessity, deals only with a very small body of information. Without consciously altering his attitude, he is forced to be extremely careful and scrupulous while copying the rare fragments available to him; he has little or no choice in a poor zone and, of course, the smaller the volume the higher his regard for the information.

However, as the chronicler moves into a rich zone of the surviving stock, the situation changes. Here comparatively large volumes of information (texts, chronicles, etc.) are to hand, and he comes across tens and hundreds of names, events, etc. Therefore, *the larger the volume of information, the lower the chronicler's regard for all its individual fragments.* In this case the chronicler has freedom of choice, and his motivations are difficult to predict. As a consequence, the amplitude correlation between the volume function of the chronicle being created by the chronicler X and the volume function of the surviving stock in the rich zone is upset. This fact is illustrated by the following example. Let the chronicler X be working at a time comparatively close to the terminal point b of the period [A, B] he is writing about (Figure 8). Here a factor such as the chronicler's judgment is particularly influential and can result in a blurred amplitude pattern of the volume graph.

We can now formulate our model in its final form and proceed to its experimental verification. This model is arbitrarily called the *principle of the amplitude correlation.*

(a) If texts X and Y are dependent, then their volume graphs vol X(t) and vol Y(t) must correlate strongly within their poor zones. On the other hand, there may be little or no amplitude correlation within their rich zones when the graphs are superimposed.

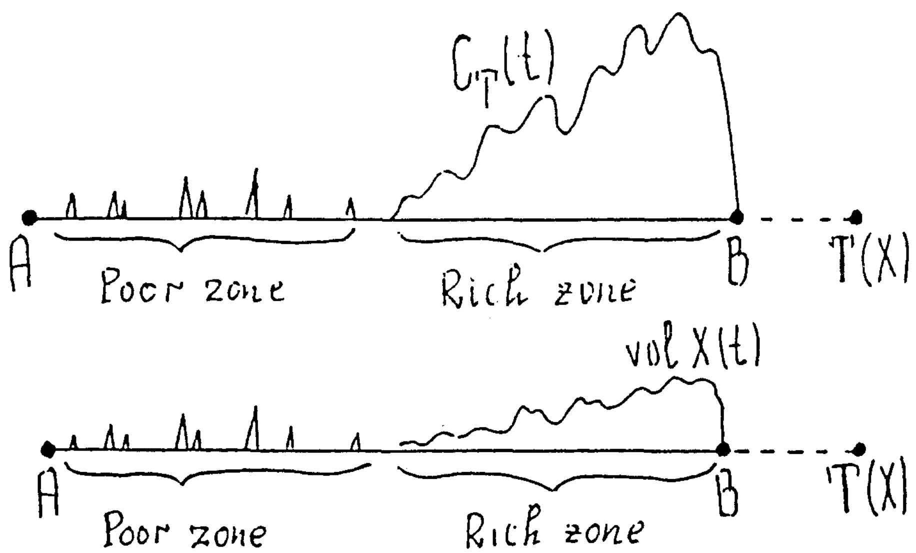

Figure 7.

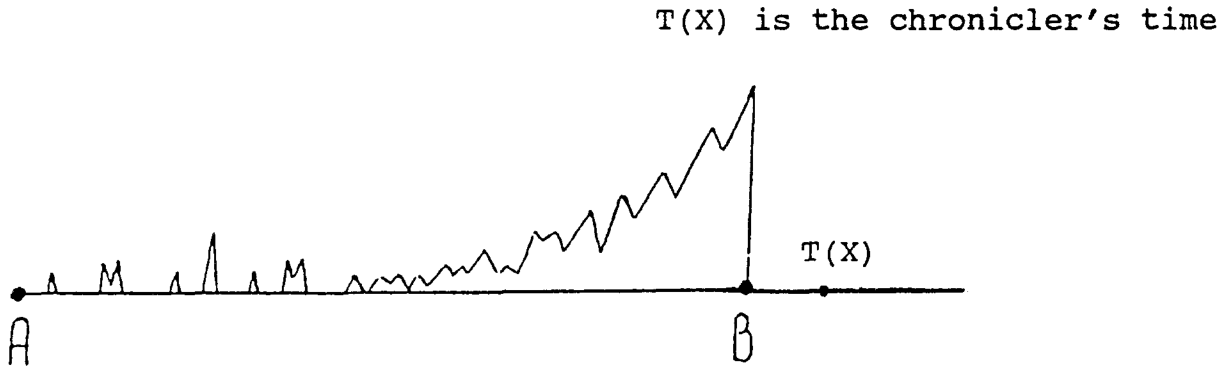

Figure 8.

(b) If texts X and Y are independent, then their volume graphs must be independent within their poor zones.

For the sake of simplicity we may assume that the volume graphs start with a poor zone and end with a rich one (see Figure 4). Then our regard-for-information principle may be paraphrased as follows: *volume graphs for dependent texts must correlate strongly at the beginning; volume graphs for independent texts must not correlate at the beginning.*

We have to provide mathematical formalization for this idea and develop particular methods for computing and estimating the correlation of two volume graphs. We shall do this with the help of the mathematical concepts listed below.

Statistical Model Description

Consider a time period [A, B] and let x be a coordinate ranging from 0 to n, where $n = B - A$ is the length of the time period of interest. Let $f(x) = \mathrm{vol}\, X(x)$ be the volume function of the chronicle X where x is a time parameter. Obviously, $x = t\text{-}A$. We denote by $\tilde{F}(x)$ a function $\Sigma_{s=1}^{x} f(s)$, i.e. essentially the integral of f from 1 to x. This function is called the *accumulated sum* (for the text X) (Kalashnikov, Rachev, Fomenko, 1986). Also useful for our purposes is the normalized accumulated sum (NAS), F(x), defined as $F(x) = \tilde{F}(x)/\mathrm{vol}\, X$, where vol X is the total volume of the text X (describing the whole period [A, B]).

A random variable ξ exhibits the Weibull-Gnedenko distribution with parameters α and λ (where $\alpha, \lambda > 0$), if the distribution function W(x) takes the form $W(x) = 1 - \exp\{-\lambda x^{\alpha}\}$, with $x \geqslant 0$ (see, for example, Galambos, 1978). Our hypothesis is that if n is large enough then the NAS F of X is described by the Weibull-Gnedenko distribution W (see Appendix A1).

We now attempt modeling the behavior of F by means of W, i.e. we shall find out just how close the function $\exp\{-\lambda x^{\alpha}\}$ approximates the behavior of $1 - F(x)$. We call α the *shape parameter* and λ the *scale parameter.* Presently we shall see that α is the more important since it is sensitive to the distribution of individual widely spaced splashes on the volume graph within a poor zone. In other words, α mainly indicates whether the texts compared are dependent or not. On the other hand, λ is primarily responsible for the text volume, serving as an indication of the richness or otherwise of the text (see Appendix A2).

We formulate our hypothesis as follows: (a) if the texts X and Y are dependent, then the corresponding parameter pairs (α_X, λ_X) and (α_Y, λ_Y) must be "close" (under the condition that they have been computed for poor text zones); (b) if the texts X and Y are independent, then (α_X, λ_X) and (α_Y, λ_Y) are "far from each other."

Historical Texts Investigated and Their Characteristics

The following texts provided the material for our study:

(1) *The Primary Russian Chronicle* (1978) is one of the most famous manuscripts in the history of Russia, and spans the period from 800 A.D. to 1200 A.D.

(2) *The Nikiforovskaya Chronicle* (see *Complete Collection of Russian Chronicles*, 1980) is of particular interest for the 600-year-long period from 850 to 1450 A.D.

(3) *The Suprasl'skaya Chronicle* (see *Complete Collection of Russian Chronicles*, 1980) covers the interval from 850 to 1450 A.D.

Chronicles (2) and (3) may be considered rather poor texts with respect to volume compared with (1). On the other hand, chronicles (2) and (3) cover a considerably longer historical period (600 years).

(4) *The Academical Chronicle* (see *Complete Collection of Russian Chronicles*, 1980) covers the interval from 1338 to 1378 A.D. With respect to volume, this text is mid-way between a poor and a rich text.

(5) *The Kholmogorskaya Chronicle* (see *Complete Collection of Russian Chronicles*, 1980) spans the interval from 850 to 1560 A.D. and includes poor as well as rich zones.

(6) *The Dvinskaya Chronicle* (see *Complete Collection of Russian Chronicles*, 1980) covers the period from 1390 to 1750 A.D. and consists of both poor and rich zones. Furthermore, in this paper, we also have analyzed texts related to ancient and medieval Rome.

(7) Titus Livius, *History of Rome* (1914), covers in particular the period from 753 B.C. to 240 B.C. The year 240 B.C. was chosen as the right terminal point of the time interval being studied in order to simplify comparing this text with others. The text begins with a poor zone and gradually becomes richer.

(8) V.S. Sergeev, *Essays on the History of Ancient Rome* (1938), covers in particular the same period of ancient Rome historically as the basic work of Livius.

(9) F. Gregorovius, *History of the City of Rome in the Middle Ages* (1900—09), covers in particular the period from 300 A.D. to 1600 A.D. (For more details concerning the texts see Appendix A3.)

The Russian chronicle texts listed above were divided into fragments covering a hundred (sometimes two hundred) years, while in the above texts on the history (ancient and medieval) of Rome, two-hundred-year fragments were isolated. Each of the fragments was then analyzed using the regression method described below. This yielded the parameters α_X and λ_X, as well as the correlation coefficient r showing how close the corresponding function $\exp\{-\lambda x^{\alpha}\}$ approximates $1 - F(x)$ where F is the NAS of the text X. These parameters were evaluated by means of linear (least squares) regression (see Kalbfleisch and Prentice, 1980) of the transformed model, $y = \ln\lambda + t\alpha$, $y = \ln[-\ln g(x)]$ and $t = \ln x$. Since all chronicles record information on an annual basis, x takes the values 1, 2, ... where one step corresponds to 2 years, and $1 - g(x)$ is the sample distribution function.

The estimated pairs $(\hat{\alpha}, \hat{\lambda})$ are depicted in Figure 9 as points on a plane with $\hat{\alpha}$ and $\hat{\lambda}$ as Cartesian coordinates.

(1) On the horizontal axis the values of $\hat{\alpha}$ from 0 to 5 are plotted. In our experiments we have not yet encountered any values of $\hat{\alpha}$ exceeding 5. On the vertical axis the values of $\hat{\lambda}$ are plotted. However, here we use a nonlinear scale: the first dashed line marks and values of $\hat{\lambda}$ from 0 to 0.0001, in which the step size is 0.00001; the next dashed line indicates the 0.0001 to 0.001 range (the step size is 0.0001), etc. Within each "band" on the graph the scale factor is an order of magnitude greater than within the preceding one. The points indicate number pairs $(\hat{\alpha}, \hat{\lambda})$ and the adjacent symbols designate the corresponding text. The texts on the history of Rome are marked with crosses. We shall next explain the legend in more detail.

The decreasing function $1 - F(x)$ in the cases listed can be closely approximated by $\exp\{-\lambda x^{\alpha}\}$, provided λ and α have been correctly determined. This follows from the last column of the table containing the values of the correlation coefficient r.

Dependent Text Comparison

We begin by comparing two *a priori* dependent texts which are equally poor in their initial parts: the Nikiforovskaya chronicle and the Suprasl'skaya chronicle. Each of them is first partitioned into the following overlapping fragments: 854—950, 960—1060, 1110—1310, 1236—1340, 1330—1432 A.D. We also isolate the interval associated with the corresponding fragments of the Nikiforovskaya and Suprasl'skaya chronicles (describing the same period) which are close to each other.

Indeed, from Figure 9, it can be noted that N1, S1 virtually coincide; N2, S2 are very close; and N3, S3 as well as N4, S4 practically coincide. By contrast, N5, S5 are far from each other. This is not surprising since they are associated with those fragments of the chronicles which describe the final period (from 1330—1432 A.D.) covered in these texts. This means that it is a rich zone

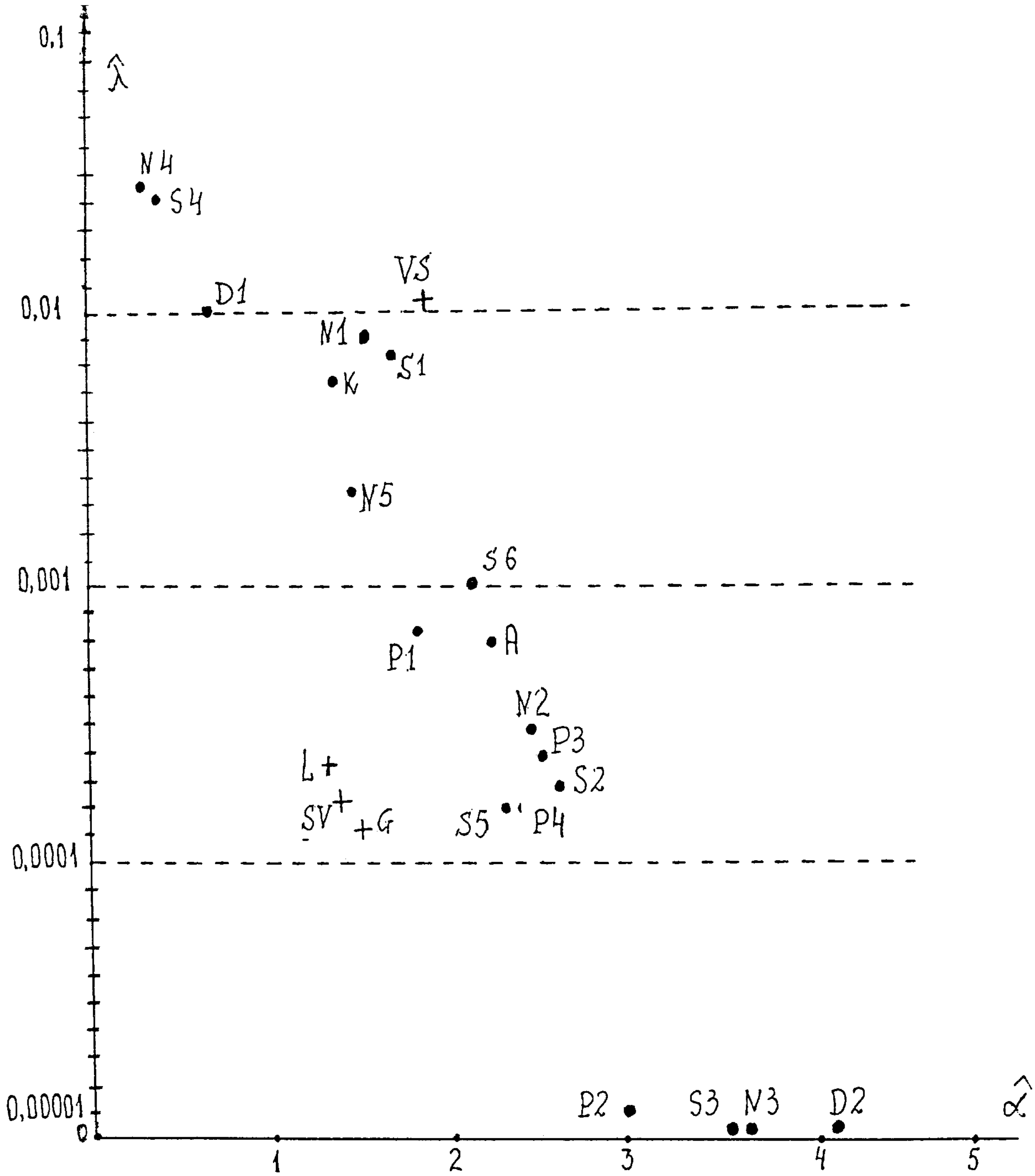

Figure 9.

Table 1. The main results of the numerical analysis, with parameters α and λ estimated using linear regression.

Legend	Chronicle (text)	Time interval described	$\hat{\alpha}$	$\hat{\lambda}$	r
P1	Russian Primary	854—950 A.D.	1.847	3.9×10^{-3}	0.953
P2	Russian Primary	913—1018 A.D.	3.003	1.6×10^{-5}	0.955
P3	Russian Primary	960—1060 A.D.	2.497	4×10^{-4}	0.956
P4	Russian Primary	998—1098 A.D.	2.378	1.3×10^{-4}	0.954
N1	Nikiforovskaya	854—950 A.D.	1.511	9.3×10^{-3}	0.966
N2	Nikiforovskaya	960—1060 A.D.	2.406	5×10^{-4}	0.917
N3	Nikiforovskaya	1110—1310 A.D.	3.685	7×10^{-9}	0.660
N4	Nikiforovskaya	1236—1340 A.D.	0.341	0.488	0.768
N5	Nikiforovskaya	1330—1432 A.D.	1.390	3.9×10^{-3}	0.953
S1	Suprasl'skaya	854—950 A.D.	1.604	8.2×10^{-3}	0.969
S2	Suprasl'skaya	960—1060 A.D.	2.584	3×10^{-4}	0.943
S3	Suprasl'skaya	1110—1310 A.D.	3.617	7.8×10^{-9}	0.656
S4	Suprasl'skaya	1236—1340 A.D.	0.405	0.384	0.808
S5	Suprasl'skaya	1330—1432 A.D.	2.354	1.6×10^{-4}	0.983
S6	Suprasl'skaya	1336—1374 A.D.	2.089	1.3×10^{-3}	0.977
A	Academical	1336—1374 A.D.	2.185	8×10^{-4}	0.960
K	Kholmogorskaya	852—946 A.D.	1.311	7.3×10^{-3}	0.960
D1	Dvinskaya	1396—1498 A.D.	0.648	0.119	0.844
D2	Dvinskaya	1500—1600 A.D.	4.060	2.2×10^{-7}	0.875
L	Titus Livius	750—510 B.C.	1.289	3.7×10^{-3}	0.979
SV	Sergeev	750—510 B.C.	1.358	2.3×10^{-3}	0.980
VS	Inverse Sergeev	510—750 B.C.	1.9	0.01	0.961

(located most likely quite close to both chroniclers), where our principle of amplitude correlation of volume functions does not necessarily hold. Figure 10 illustrates the volume functions of the Nikiforovskaya and Suprasl'skaya chronicles on the whole interval from 850 to 1450 A.D. A poor and a rich zone are indicated, the former covering the period from 850 to 1330 A.D. and the latter covering the final text period from 1330 to 1450 (or from 1330 to 1432) A.D. The Nikoforovskaya chronicle terminates somewhat earlier (in 1432 A.D.). The amplitude correlation of the volume functions is directly visible here (the graphs are closely similar to each other), and it is therefore not surprising that the dependence of the texts was confirmed numerically.

The next pair of compared texts is of particular interest since they are known to be dependent. However, one of them is poor whilst the other is rich. These are the Russian Primary Chronicle and Nikiforovskaya chronicle (or, what is essentially the same, the Suprasl'skaya chronicle). Figure 10 illustrates the volume graph of the Russian Primary Chronicle. This graph is shorter than the Nikiforovskaya and Suprasl'skaya chronicles but its amplitudes are considerably larger. Note that there is no pronounced visual relationship of the amplitudes. The Russian Primary Chronicle is subdivided into the following fragments 854—950, 918—1018, 960—1060, 998—1098 A.D. It may be recalled that when we selected the left end of the interval [A, B] we attempted to align the first non-zero value of the volume function with A or at least to bring them close. Point P1 associated with the Russian Primary Chronicle (854—950 A.D.) seems to lie on the (α, λ) plane of Figure 9 quite far from virtually coinciding points N1 and S1. These two points correspond to the fragments of the Nikiforovskaya and Suprasl'skaya chronicles where the same period from 854 to 950 A.D. has been described. It is worth remembering here that the principal parameter characterizing the "shape" of the volume function is α, (when λ is the scale parameter).

Comparing the values of $\hat{\alpha}$ for P1 and the pair N1, S1 (by projecting them onto the horizontal

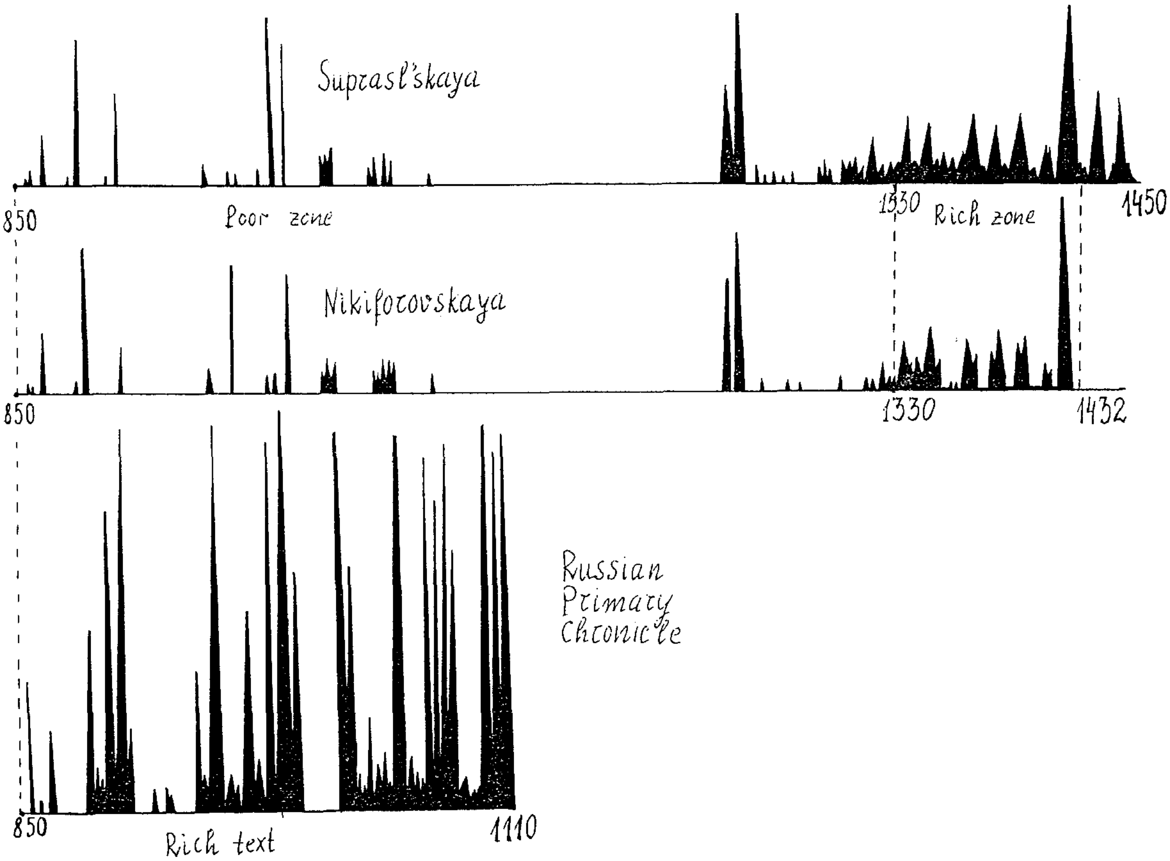

Figure 10.

axis) we notice that the three values are quite close. Therefore, the rich text P1 and the poor texts S1 and N1 are indeed dependent. The corresponding values of the scale parameter λ differ as would be expected since λ can "feel" if the text is rich or poor (see also Appendices A4, A5).

Analysis of the data enables us to infer a relationship between α and time, which we shall describe in the following section.

An Increase in α with Time for Individual Texts

If we consider texts over the whole interval from the 9th to the 17th centuries A.D., then this trend does not reveal itself distinctly enough in Figure 9. However, the situation becomes clearer if we restrict our analysis to reliably dated texts beginning with 1200 A.D. or so, or even closer to modern times. Furthermore, we divide the plane of Figure 9 into four bands according to different step sizes for $\hat{\lambda}$. We compare the locations of the points confined within one band. We start with the uppermost band, lower-bounded by the horizontal $\hat{\lambda} = 0.01$. In the left part of Figure 9 lie points N4 and S4 (1236—1340 A.D.) and to their right, i.e. having a larger value of $\hat{\alpha}$, lies D1 (1396—1498). Thus for all three points within band 4, $\hat{\alpha}$ does increase with time. Within band 3 (between 0.001 and 0.01) we have two points, N5 and S6, but they are difficult to compare since the period 1336—1374 (for S6) is significantly shorter than the period 1330—1432 (for N5). On the other hand, the beginning of S6 lies to the right of the beginning of N5, which does agree with the increase in α (see Figure 9). Within band 2 (between 0.0001 and 0.001) we again have only two points, A

(1336—1374) and P4 (1330—1432), but here the values of $\hat{\alpha}$ practically coincide (although, formally, the order is reversed). Finally, within band 1 (between 0 and 0.0001) we have three points, S3 (1110—1310), N3 (1110—1310) and D2 (1500—2600). The first two points essentially coincide. Here $\hat{\alpha}$ undoubtedly increases with time (D2 lies to the right of S3 and N3).

Increase in α with Time for Groups of Texts Belonging to the XIII—XVIth A.D.

The relative uncertainty of the general trend in the case of individual texts is due to the fact that we have attempted to evaluate rather rough quantities with too great a precision. Therefore it would be more natural to consider not individual texts, but groups of texts associated with approximately the same period (say, 50 or 100 years long) and then to compare the averaged values of $\hat{\alpha}$ for these longer periods (text groups). We shall deal only with reliably dated texts beginning with 1200 A.D. or so (that is, closer to modern times). In Figure 11 the same points as were used in Figure 9 for representing texts on the $(\hat{\alpha}, \hat{\lambda})$ plane are combined in groups corresponding to nearly the same period.

Group (1236—1340) — two texts N4 and S4.
Group (1330—1430) — four texts N5, S6, A, S5.
Group (1500—1600) — one text D2.

It is evident from Figure 11 that the consecutive groups are positioned from left to right, which corresponds to an increase in α with time. The only exception is point D1 (1396—1498) that lies close to group (1236—1340). Thus we can see that such a "consolidation" leads to a more pronounced increase in $\hat{\alpha}$ with time (although here, too, additional research is required).

Increase in α with Time for Groups of Texts Belonging to 9th—13th Centuries A.D.

The Russian chronicles belonging to the 9th—13th centuries A.D. can be combined into groups of texts describing nearby time periods, namely:

Group (854—950) — four texts N1, K, S1, P1.
Group (918—1098) — five texts N2, S2, P2, P3, P4.
Group (1110—1310) — two texts S3, N3.

Figure 11 clearly shows that the consecutive groups are positioned from left to right, which again implies an increase in α with time. Therefore, for the texts of the 9th to the 12th centuries as well as for the texts of the 12th to the 16th centuries the parameter α on average increases gradually with time, which provides evidence for our hypothesis.

Statistical Shifts by about 330—360 or 400 Years

Figure 11 shows an interesting phenomenon: the text group (918—1098) lies close to the text group (1330—1430). Comparison of these periods (say, in terms of the final points) results in the shift by about 330 (or 360) years. This means that both groups are characterized by approximately the same values of α. A similar statement is true for two other groups of texts of the 9th to the 13th centuries. Indeed, from Figure 11 we can see that group (854—950) lies on the $(\hat{\alpha}, \hat{\lambda})$-plane between group (1236—1340) and group (1330—1430), which also yields a shift of about 380—390 years or 480 years. Finally, the third group of the 9th to the 12th centuries, namely group (1110—1310), lies close to group (1500—1600), which results in the shift by about 400 years. Note also that this system of shifts quite naturally translates into one "rigid" shift of the 9th—12th group onto the 9th—16th group, i.e. by 300—400 years upward. As a result each of the three groups (comprising the 9th—13th group) lies beside "its own" group from the 13th—16th group. This phenomenon still needs to be examined more closely. Any specific conclusions would be premature at this point.

Statistical Shift by about 1050 years for Roman Texts

Statistical dependence between Livius' and Sergeev's texts and the text of Gregorovius has been established (see Fomenko, 1980, 1985a). It is of interest to verify this conclusion by means of the amplitude correlation principle. Figure 9 shows that the point G depicting the book by F. Gregorovius (1900), where the period of 300 to 540 A.D. is described, essentially coincides with two points L and SV depicting Livius' and

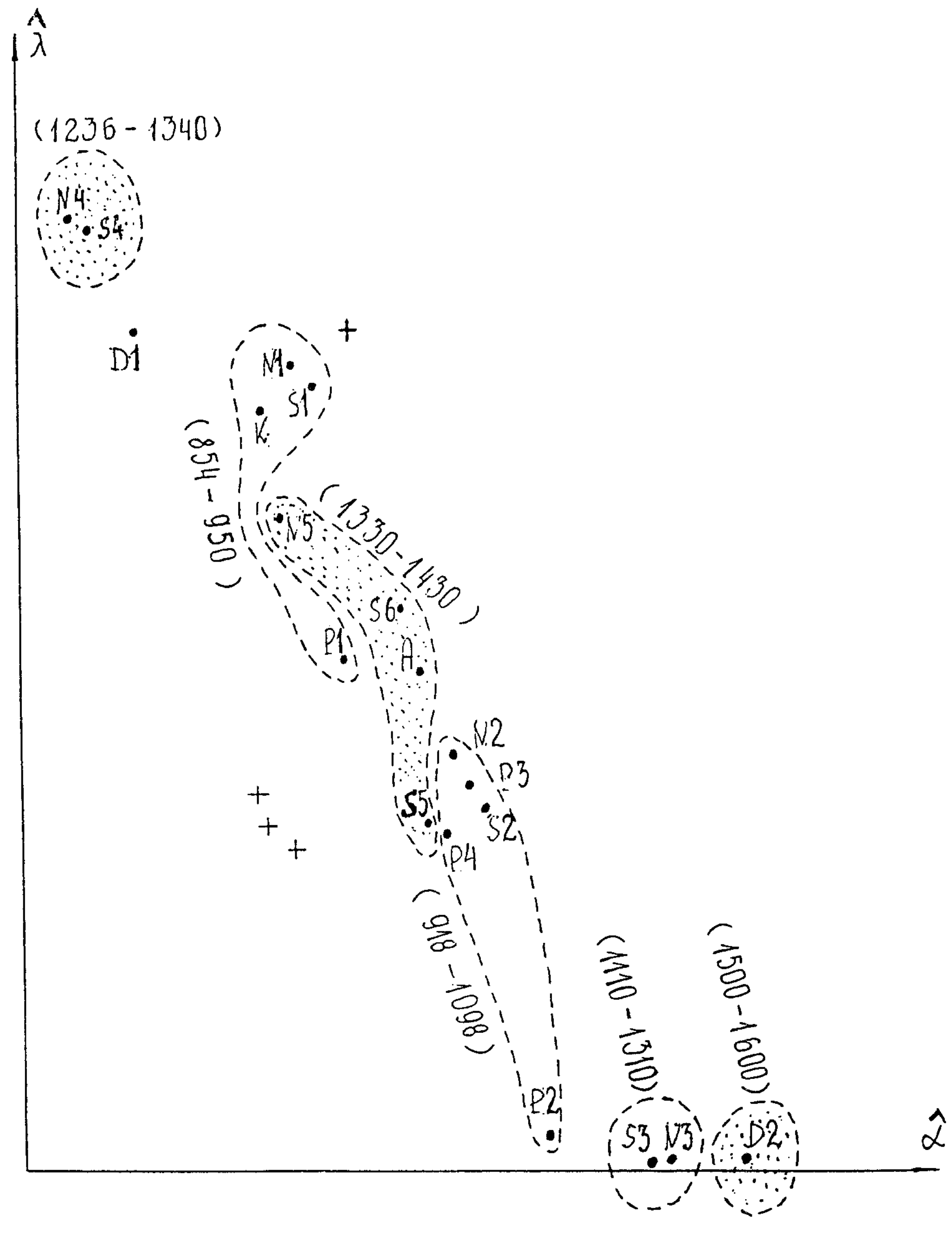

Figure 11.

Sergeev's texts, where the period from 750 to 510 B.C. is described. This indicates a clear-cut dependence of the three texts. The corresponding volume functions are illustrated in Figures 12 and 13.

Note that the periods shown in Figures 12 and 13 are longer than those we have analyzed. The point is that the volume function of the three texts correlate on the whole interval 753—236 B.C. and 300—816 A.D. In Figure 13 Livius' text that terminated in 294 B.C. is extended by Sergeev's text up to 236 B.D. Comparing the dates results in a shift by some 1050 years or so (see also Appendices A6—A7).

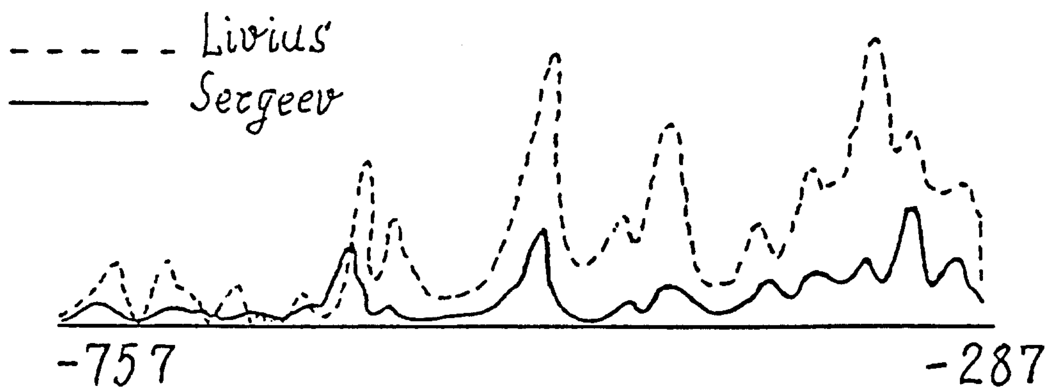

Figure 12.

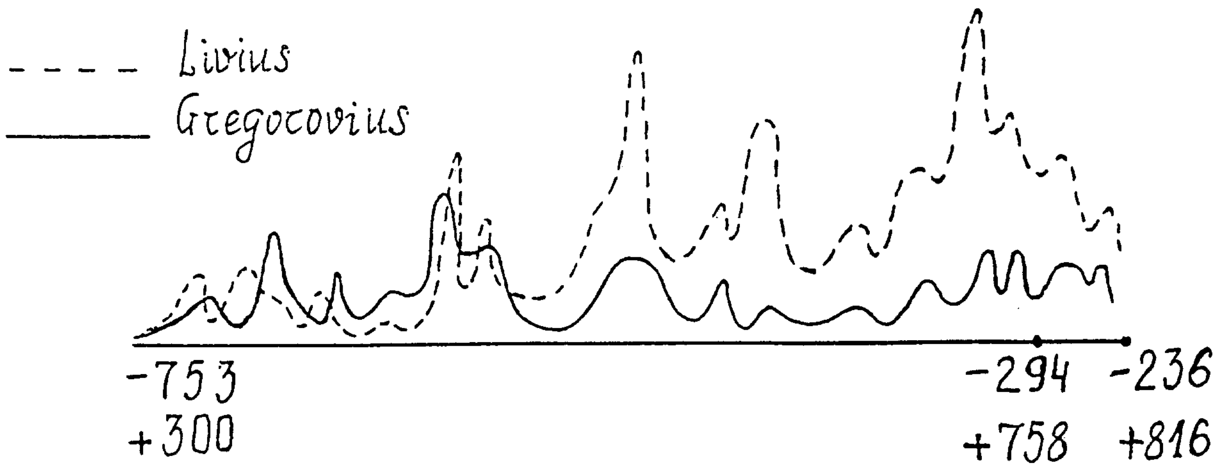

Figure 13.

Summary

The main results of the present study may be summarized as follows:

(1) A new empirico-statistical model has been formulated that enables recognition of dependent and independent texts. The regard-for-information principle and amplitude correlation principle have been formulated.

(2) The model and both principles have been verified in the course of numerical studies involving specific historical texts. The texts were reliably and positively dated. The model and the two principles have been proved valid.

(3) The results of the verification enable introduction of a methodology for the recognition of dependent and independent texts.

(4) The analysis of historical texts resulted in specific conclusions of a statistical nature:

(a) The decreasing function $1 - F(x)$, where F is the normalized accumulated sum of historical text volume, may be closely approximated by the function $\exp(-\lambda x^{\alpha})$ with appropriately chosen parameters α and λ.

(b) If the texts X and Y are dependent, the corresponding points (α_X, λ_X) and (α_Y, λ_Y) on the (α, λ) plane are close to each other (see also A4).

(c) If the texts X and Y are independent, the corresponding points (α_X, λ_X) and (α_Y, λ_Y) are far from each other (see also A5).

(d) The parameter α (and sometimes λ) characterizes the whole group of texts describing the events of a given historical epoch, i.e. the pair (α,

λ) is the "invariant of the epoch" or, more precisely, of its surviving information stock (see also A6).

(e) Our results point to the possible existence of shifts by 300—400, 1000 and 1800 years (see also A7).

Acknowledgement
The authors are grateful to the referees whose comments and suggestions for reorganization of the material resulted in a better presentation.

Appendix A1. The Hypothesis for Weibull-Gnedenko Distribution

Analyzing experimental data one can see that the "poor" X and the "rich" Y are close to each other only at the beginning of the interval under consideration. This is due to the fact that with the passage of time the rich text contains more references to "closer" events, while the references to the beginning of the period under consideration disappear.

Our model for comparing texts is based on their proximity during the "initial interval" of the period being investigated.

Let $F_0 = F_{X_0}$ be the normalized accumulated sum (NAS) that corresponds to an "ideal" text X_0, i.e. the text that would have been written by an absolutely objective and absolutely informed chronicler. Let $g_0 = 1 - F_0$. A "real-life" chronicler writes the text X with a NAS F_X, and his text is a "transformation" of the ideal text X_0. We shall assume that this transformation is characterized by the proximity function

$$g_X(x) = 1 - F_X(x) \approx g_0^n(x/a_n), \qquad (1)$$

where n is a sufficiently large integer and $\{a_n\}$ is a sequence of non-negative constants. The rationale of the assumption (1) is as follows. If the chronicler were "absolutely objective" but not "absolutely informed," then, assuming a uniform information loss, the volume function vol X(t) of his text X would be "uniformly lower" than the volume function vol $X_0(t)$ of the ideal text. Thus we can write $F_0 \approx F_{X_0}$, whence $g_0 \approx g_X$. If this situation corresponds to a rich text Y, then for a poor text we have $F_X(x) \geqslant F_Y(x) \approx F_0(x)$, i.e.

$$g_X(x) \leqslant g_Y(x) \approx g_0(x) \qquad (2)$$

and for the values of x close to zero we must have

$$g_X(x) \approx g_Y(x) \approx g_0(x) \qquad (3)$$

i.e. the real-life text (whether rich or poor) is close to the ideal text in the neighborhood of zero. This is due to the fact that with time the chronicler introduces more and more "noise." On the other hand, according to the regard-for-information principle he includes in his text the valuable information whose volume is small if one considers the large length of the period under investigation. Hence, the accuracy of the chronicler reproducing the survived information is inversely proportional to the volume of information available to him.

Besides, (1) permits another, probabilistic interpretation. If $\xi_1, \xi_2, \ldots,$ is a sequence of independent random variables with a common distribution function (DF), F_0, then

$$g_0^n(x/a_n) = \mathrm{Prob}(a_n \min(\xi_1, \ldots, \xi_n) > x). \qquad (4)$$

It is well-known (see for example, theorem 2.1.5, Galambos 1978), that if

$$\lim_{u \to 0} F_0(ux)/F_0(u) = x^\alpha \quad (\alpha > 0) \qquad (5)$$

then as $n \to \infty$

$$g_0^n(x/a_n) \to 1 - \varphi_{\lambda,\alpha}(x) = \begin{cases} 1 & x \leqslant 0 \\ \exp\{-\lambda x^\alpha\} & x > 0 \end{cases} \qquad (6)$$

where $\varphi_{\lambda,\alpha}$ is the Weibull-Gnedenko distribution and the constants $a_n = a_n(\alpha, \lambda)$ are subject to

$$1/a_n = \sup\{x: F_0(x) \leqslant 1/n\} \qquad (7)$$

The relations (1)—(7) lead us to the hypothesis that $F_X(x)$ (the NAS of X) is close to the Weibull-Gnedenko distribution for x small. In other words, $1 - F(x)$ must behave (at least in the initial poor text zone) as $\exp\{-\lambda x^\alpha\}$.

The answer of whether our approximation is good or not, also depends on *the choice of a metric for comparing texts*. Let X be a given text. The examination of rich and poor texts reveals that usual criteria for assessing proximity of two NAS functions F_X and F_Y are not adequate.

The reason is that the NAS of a rich text lies below the NAS of a poor text. For example, if κ is the Kantorovich metric in the DFs space on [0,

∞], i.e., $\kappa = \kappa(F, G) = \int_0^\infty |F(x) - G(x)|dx$, then the deviation $\kappa(F_X, F_Y)$ may be greater than the value of $\kappa(F_X, F_Z)$ for independent texts X and Y. The reason for the quantity $\kappa(F_X, F_Z)$ being smaller than $\kappa(F_X, F_Y)$ is as follows. Let X and Z be two independent poor texts, then even if the steps of F_X and F_Z may not coincide, the behavior of these two functions may be essentially identical due to the fact that both texts are poor. Therefore, the metric κ tends to "distinguish" between a poor and a rich text better than between an independent and a depended text. This metric represents the class of "weak" metrics, i.e. if DFs are concentrated on a compact set (which is true for NASs), then κ introduces the weak convergence (see, for instance Rachev, 1986a, 1986b). Within the framework of the theory of probability metrics (see Rachev, 1988, 1989) the problem of constructing a metric that effectively "distinguishes" dependent texts from independent ones leads naturally to the study of topologically "hard" probability metrics such as $\mu(F, G) = \Sigma_{x \geqslant 0} |(F(x + 0) - F(x - 0)) - (G(x + 0) - G(x - 0))|$, which in essence reduces to the maxima correlation principle (see also Kalashmikov, Rachev and Fomenko, 1986, where different approaches to this problem were examined). The rate of convergence in (6) as $n \to \infty$ was examined in Smith (1982), Resnick (1987), Omey and Rachev (1988), deHaan and Rachev (1989).

Appendix A2. The Shape Parameter α is the Same for Dependent Texts

If X and Y are dependent poor texts, then the parameter pair (λ_1, α_1) must be proximal to the pair (λ_2, α_2) in the Euclidean metric on the plane. However, if X is poor and Y is rich, then even though X and Y are dependent, we cannot hope for $(\lambda_1, \alpha_1) \approx (\lambda_2, \alpha_2)$, because if this approximate equation holds, then $\kappa(F_X, F_Y)$ should be close to zero, which is actually not true as we have already pointed out in A1. It is quite natural that in such a situation the "scale" parameter λ_1 and λ_2 of the NASs F_X and F_Y, respectively, will differ from one another, but the "shape" parameters α_1 and α_2 will nevertheless be close. Indeed, according to (5) and the proximity of g_X, g_Y, in the neighborhood of zero (see (3)), it follows that the difference in α_1 and α_2 makes (3) invalid. Since (5) is independent of λ, then it is α that is fully responsible for the behavior of rich and poor texts in the beginning. Hence if X and Y are dependent texts, then the approximations (8) and (9) lead to the proximity of α_1 and α_2.

A3. The Historical Texts

Among the texts (1–9) there are some *a priori* dependent and some *a priori* independent.

(a) *A priori* dependent are the Nikiforovskaya and Suprasl'skaya chronicles (both can be considered equally poor). Dependence of these texts has been verified using a method based on the maxima correlation principle (see Fomenko, 1980, 1981a, 1985a).

(b) *A priori* dependent are Livius' and V. S. Sergeev's texts because the latter is based mainly on the books of T. Livius that are recognized as one of the most important sources of information on the history of ancient Rome. Dependence of these texts has also been verified where the correlation of the volume function maxima for the two texts was established.

(c) *A priori* dependent are the Russian Primary Chronicle and the Nikiforovskaya chronicle (and hence the Russian Primary Chronicle and the Suprasl'skaya chronicle). Dependence of these three texts was also verified in view of the correlation of their volume function maxima.

Other examples of dependent texts will be given along with the analysis of the numerical results. Now we turn to *a priori* independent texts.

(d) *A priori* independent are the part of the Dvinskaya chronicles covering the 14th A.D. and its part related to the 15th A.D. It is clear that two consecutive chronicle fragments (each a hundred years long) describing well-known and well-documented events in the interval from the 15th to the 16th A.D. are independent (in all senses).

(e) *A priori* independent are Sergeev's text and the "inverse Sergeev" text. Actually, independent texts are easy to obtain if one is interested only in the text volume function. To do this one has to "reverse" the text, i.e. to keep the partitioning into chapters and to enumerate them in reverse order (whereby the last chapter becomes the first, etc.). Superimposing the "original" text and the "inverse" text obviously results in independent (in all senses) texts.

Here an important observation is due. One should not think that *a priori* dependent texts are essentially identical from the viewpoint of their contents. This is not so. For example, the author of the Nikiforovskaya chronicle has not mentioned the events of 977 A.D. while the author of the Suprasl'skaya chronicle devoted 4 lines of text to this year (even though these two texts are *a priori* dependent). Here is another example: the Suprasl'skaya chronicle records the marriage of Prince Alexander Nevski in 1233, while the Nikiforovskaya chronicle contains no mention of this event. Therefore, the intrinsic regularities discussed in the present paper govern chroniclers without them being aware of it. These regularities are revealed only in large-scale numerical experiments with large volumes of data involved in the study.

A4. More on Dependent Text Comparison

We continue comparison of texts depicted on Figure 9. We compare three points: P3, N2 and S2, i.e. the Russian Primary Chronicle, Nikiforovskaya and Suprasl'skaya chronicles, respectively. All three deal with the same period (960—1060 A.D.). As can be seen from Figure 9, the three points practically coincide, which indicates strong dependence of these texts. It would be interesting to know what will happen if we "stir" one of these texts slightly, e.g. shift the time interval being described. To this end we "cut out" from the Russian Primary Chronicle the fragment P2 that relates to the period form 918 to 1018 A.D., i.e we step 40 years down as compared to the interval from 960 to 1060 A.D. One might expect the point (α, λ) depicting the corresponding portion of the Russian Primary Chronicle to move somewhere since the text 918—1018 A.D., although close to the texts of 960—1060 A.D., still becomes more and more independent nominally. Indeed, Figure 9 shows that the point (α, λ) associated with P2 (i.e. 918—1018 A.D.) has moved somewhat to the right and downward with respect to the three neighboring points P3, N2, S2. Therefore, the choice of the parameters α and λ was "right" in that they are very sensitive to the time interval variation that drives the points (α, λ) associated with independent texts apart (which is quite correct). However, the points have not moved too far from each other since the time interval shift was not too big. This means that as the epoch changes, the corresponding pair (α, λ) varies, in a sense, "continuously."

Finally, we shall compare P4 (the Russian Primary Chronicle, 998—1060 A.D.) with N2, S2 (the Nikiforovskaya and Suprasl'skaya chronicles, 960—1060 A.D.). These points are associated with the texts that are close in time and, therefore, dependent. Indeed, in Figure 9 these three points are again close to each other, thereby confirming the dependence of the texts. A slight variation in the time interval (960—1060 and 998—1098) affects the dependence only little.

Thereby we have essentially exhausted the Russian Primary Chronicle that terminates at 1110 A.D. The dependence of three texts (the Russian Primary Chronicle, Suprasl'skaya and Nikiforovskaya chronicles) bears out very well. This means that our amplitude correlation principle is indeed confirmed for dependent texts (in poor zones). Even more, in some cases this principle is valid for rich zones (e.g. for the last part of the Russian Primary Chronicle), which we could not have hoped for because of the model that has been adopted.

Now we shall compare some more *a priori* dependent texts. The Kholmogorskaya chronicle (point K in Figure 9) describes the period from 852 to 946 A.D. that virtually coincides with the period from 854 to 950 A.D. referred to in the Nikoforovskaya and Suprasl'skaya chronicles (points N1 and S1 in Figure 9). Therefore, the three texts are dependent (in the formal sense, i.e. they describe the same period in the history of essentially the same region). Indeed, the corresponding points K, N1 and S1 are very close in Figure 9, providing further support to the amplitude correlation principle (for poor zones).

The Academical chronicle spans the periods from 1336 to 1374 A.D. and therefore is formally dependent with the portion of the Suprasl'skaya chronicle that describes the same period. In Figure 9 the corresponding points A and S6 lie close to each other on the $(\hat{\alpha}, \hat{\lambda})$-plane. Again this lends support to the validity of the amplitude correlation principle.

Livius' text is dependent upon Sergeev's text for the obvious reason that the latter was based on the information provided by the former. Livius' text is considered to be the primary source of information on the history of ancient Rome (the period from 750 to 510 B.C. included). The corresponding points L and SV are close (see Figure 9), which confirms the amplitude correlation principle. Now we have to verify the second part of the amplitude correlation principle, namely; there should be no correlation for *a priori* independent texts (see A5).

A5. Independent Text Comparison

Consider a number of independent text pairs. To clear away all doubts as to text independence we restrict ourselves either to the texts dealing with time periods after about 1300 A.D. (and closer to our times) or to the ones whose independence has been reliably proved by different techniques (including statistical). For example, let us divide the Dvinskaya chronicle into two parts: 1396—1498 A.D. and 1500—1600 A.D. They are obviously independent (see above). The corresponding points D1 and D2 in Figure 9 are indeed quite far from each other (at diametrically opposite ends of the region where the experimental results are located). Thus the *a priori* independence of the texts compared is clearly illustrated by the difference in the parameters α, λ. The next example is Sergeev's text (point SV in Figure 9) and the "inverse Sergeev" (point VS in Figure 9). These points are widely spaced due to text independence. Consider the Nikoforovskaya chronicle. Two parts of this text, 1110—1310 and 1236—1340 A.D., are *a priori* independent since they belong to well-dated epochs and overlap in time only little (26 years, which is small as compared to the 100-year-long interval). As can be seen from Figure 9, the corresponding points N3 and N4 are widely separated due to obvious independence of the texts, and so on.

A6. The Parameter α (and, in Part, the Paramenter λ) are the Epoch "Invariants"

Our results permit us to formulate the following hypothesis: each horizontal epoch (about 100 or 200 years long) is characterized by its "own" values of the parameter α (and, possibly λ). This hypothesis is essentially a paraphrase of the phenomenon discussed above, namely that of increase in α with time. Intuitively, we may substantiate this hypothesis by observing that each epoch is characterized by its "own" surviving stock of information (on past events). This stock grows gradually as we move from left to right along the time axis (the conditions under which documents are stored get better, the document loss rate gets lower). Put another way, the hypothesis consists in that the points $(\hat{\alpha}, \hat{\lambda})$ separate not only individual texts, but different epochs (100 to 200 years long) as well. When describing one epoch, different chroniclers make use of approximately the same surviving stock. As a result, all chronicles describing the same period "on average" behave similarly, i.e. exhibit close values of α and λ.

A7. Statistical Analysis of Some Medieval and Ancient Texts. Statistical Shift by 1800 Years

The methods described above were also applied to some texts whose dependence has been established by Fomenko (1980, 1981a, 1985a), using the maxima correlation principle. Here we shall verify the dependence of these texts from the viewpoint of the amplitude correlation principle. The volume functions of the Holy Roman emperors' "biographies" in the 10th—13th centuries A.D. were calculated by Fomenko (1985a). By a "biography," we understand that part of the text, which describes the events during the emporer's (tzar's, etc.) reign. If the description began at a time not clearly specified in the text, we take as the start of the "biography" the first mention in the text of the corresponding ruler in connection with the events of the epoch being described. As the first text, the monograph by Kohlrausch (1986) was selected and then the book by Fedorova (1985) was analyzed. A part of the latter book describes the historical period from 919 to 1300 A.D. This pair of *a priori* dependent texts was found to possess volume functions strongly correlating from the viewpoint of our amplitude correlation principle, namely: for Kohlrausch (1986), $\lambda = 1.455 \times 10^{-3}$, $\alpha = 1.457$, r = 0.930; for Fedorova (1985); $\lambda = 1.165 \times 10^{-3}$, $\alpha = 1.497$, r = 0.922. Then this pair of texts was compared with the Judaic kings' "biographies" (in terms of volume functions) that belong to the 8th—6th B.C. (in traditional chronology). For volume calculation the Bible was used. Dependence between the resulting volume function and the two *a priori* dependent functions was first established by Fomenko (1980, 1983). We have also found them dependent from the viewpoint of the amplitude correlation principle. The statistical shift amounts to about 1800 years.

Then we checked the dependence of the volume function for Herodotus' History (560—464 B.D.) and the volume function for Gregorovius' History of the City of Athens in the Middle Ages, (1254—1350 B.C.). The two historical periods may be obtained from one another through a rigid shift about 1800 years. Volume functions were calculated for the "biographies" of such rules as Cresus, Cyrus I, Cambyses, Darius I, Zerxes (according to Herodotus) and Manfred, Charles of Anjou, Charles II of Naples, Frederick II of Sicilly, Duke Walther II de Brenne (according to Gregorovius). These volumes were distributed over the time interval allowing for the rulers' regin durations. As a result we have discovered a correlation of the volume function amplitudes, namely for Herodotus

$$\lambda = 4.977 \times 10 - 4, \quad \alpha = 2.126, \quad r = 0.964$$

while for Gregorovius

$$\lambda = 1.327 \times 10 - 3, \quad \alpha = 1.827, \quad r = 0.967.$$

References

Complete Collection of Russian Chronicles. Vol. 35. Moscow: Nauka, 1980. (In Russian.)

Complete Collection of Russian Chronicles. Vol. 33. Leningrad: Nauka, 1977. (In Russian.)

deHaan, L. and S. T. Rachev. "Estimates of the Rate of Convergence for Max-Stable Processes." *Annals of Probability*, 17 (1989), 651—77.

Fedorov, E. F. *Famous Italian Cities.* Moscow: Moscow Univ. Press, 1985.

Fedorov, V. V. and A. T. Fomenko. "Statistical Estimation of Chronological Nearness of Historical Texts." *Journal of Soviet Mathematics*, 32, 6 (1986), 668—75.

Fomenko, A. T. "Certain Statistical Regularities of Information Density Distribution in Texts with Scale." *Semiotika i Informatika.* Moscow. Vol. 15 (1980), 99—124. (In Russian.)

Fomenko, A. T. "Some New Empirico-Statistical Methods of Dating and the Analysis of present Global Chronology." (1981) (English translation is catalogued in the British Library, Department of Printed Books, Cup. 918/87).

Fomenko, A. T. "A Method for Duplicate Recognition and Some Applications." *Doklady Akademii Nauk*, 258, 6 (1981), 1326—30. (In Russian.)

Fomenko, A. T. "New Empirico-Statistical Methods for Ordering Texts and Applications to Dating Problems." *Doklady Akademii Nauk*, 268, 6 (1983), 1322—27. (In Russian.)

Fomenko, A. T. "New Empirical and Statistical Methods for Recognizing Parallels and Duplicate Dating." *Problemy Ustoichivosti Stokhasticheskikh Modelei.* Systems Research Institute, Moscow, Workshop Proceedings, 1984, pp. 154—77. (In Russian.)

Fomenko, A. T. "Informative Functions and Related Statistical Regularities." Statistika-Veroyatnost', Ekonomik Ser, *Uchenye Zapiski po Statistike.* Moscow: Nauka, 1985, pp. 335—42. (In Russian.)

Fomenko, A. T. "Duplicates in Mixed Sequences and Frequency Decay Principle." *Abstracts of Communications of the Fourth International Vilnius Conference on Probability Theory and Mathematical Statistics.* Institute of Mathematics and Cybernetics of AN LSSR, 3, 1985, pp. 246—48. (In Russian.)

Galambos, J. *The Asymptotic Theory of Extreme Order Statistics.* New York: Wiley, 1978.

Gregorovius, F. *History of the City of Rome in the Middle Ages.* London: G. Bell & Sons, 1900—09.

Kalashnikov, V. V., S. T. Rachev and A. T. Fomenko. "New Methods for Comparing Volume Functions of Historical Texts." *Problemy ustoichivosti stokhasticheskikh modelei.* Systems Research Institute, Moscow, Workshop Proceedings, 1986, pp. 33—45. (In Russian.)

Kalbfleisch, J. D. and R. L. Prentice. *The Statistical Analysis of Failure Data.* New York: John Wiley, 1980.

Kohlrausch, F. A. *History of Germany, from the Earliest Period to the Present Time.* New York: D. Appleton and Co., 1896.

Morozov, N. A. *Christ.* Volumes 1—7. Moscow-Leningrad: Gosizdat, 1924—1932. (In Russian.)

Morozova, L. E. "Application of Quantitative Methods to the Analysis of the So-called Philaret's Manuscript — The Literary Moment of the Troubled Times." *Mathematischeskie Metody i EVM v Istorischekikh Issledovaniyakh.* Moscow: Nauka, 1985, pp. 182—203.

Omey, E. and S. Rachev. "On the Rate of Convergence in Extreme Value Theory." *Theorja Verojatnostei i Primenenia*, 3 (1988), 601—07. (To appear also in *Theory Probability and its Applications.*)

Rachev, S. T. "On a Class of Minimal Functionals on a Space of Probability Measures." *Theory Probability and its Applications*, 29 (1984), 41—49.

Rachev, S. T. "The Monge-Kantorovich Mass Transference Problem and its Statistic Applications." *Theory Probability and its Applications*, 29 (1984), 647—76.

Rachev, S. T. "The Stability of Stochastic Models." *Applied Probability Newsletter*, 12, 2 (1988), 3—4.

Rachev, S. T. "The Problem of Stability in Queueing Theory." *Queueing Systems Theory and Applications*, 4 (1989), 287—318.

Resnick, S. I. *Extreme Values, Regular Variation, and Point Processes.* New York: Springer-Verlag, 1987.

Sergeev, V. S. *Essays on the History of Ancient Rome.* Moscow, 1938. (In Russian.)

Smith, R. L. "Uniform Rates of Convergence in Extreme-Value Theory." *Advances in Applied Probability,* 14 (1982), 600—22.

The Primary Russian Chronicle. Moscow: Khud. Lit. Press, 1978. (In Russian.)

Titus Livius. *Works.* Cambridge, London: Heinemann, Harvard University Press, 1914.