Figure 7. *Histogram* f_2 *for* (a) *list P (popes' names),* (b) *list H (popes' nationalities).*

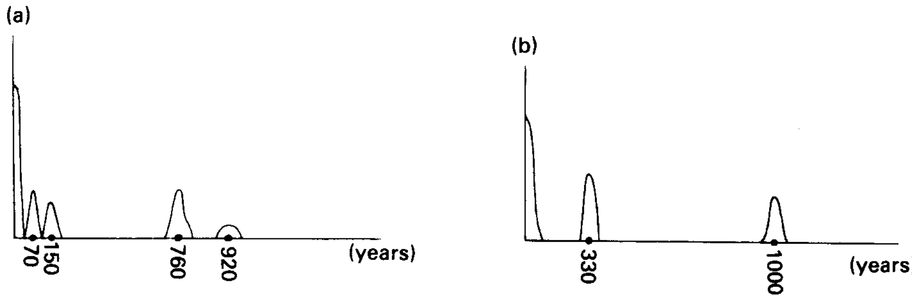

Figure 8. *Histogram* f_2^C *for the list H (popes' nationalities)*; (a) $C = X_{51}, X_{52}, \ldots, X_{80}$; (b) $C = X_{21}, X_{22}, \ldots, X_{50}$.

Besides, we observe an exceptionally sharp, four-fold peak near to the origin. The shifts through 330, 400, 760, 850, 960, 1050 and 1400 years are also explicit.

The histogram f_2 for H in Fig. 7(b) supplies much less information, and contains two sharp peaks about the origin and 600–640 years as well as two weaker ones around 330 and 450 years.

Take the probabilistic scheme from § 8. Let C be a certain subset of the list chapters, namely, $C = \{X_{i_1}, \ldots, X_{i_l}\}$. We will say that two names u_i, u_j from C are of the same age ($u_i \bumpeq u_j$) if they were 'born' in one of its chapters. We will call u_i and u_j conjugate in C ($u_i \bumpeq u_j$) if they were mentioned in one of its chapters, and write $a_i \bumpeq a_j$, or $a_i \bumpeq a_j$, if the corresponding relation is valid for the two entries in X as names from I.

Defining the event

$$A_C = \{\omega : a_{(1)} \bumpeq a_{(2)}\}, \quad B_C = \{\omega : a_{(1)} \bumpeq a_{(2)}\}, \quad \omega = (a_{(1)}, a_{(2)}),$$

we consider the frequency histograms for the names related in C as in § 8, namely,

$$f_2^C(j) = P_{A_C}(\xi_1 = j) = P(\xi_2^C = j), \quad f_3^C(j) = P_{B_C}(\xi_1 = j) = P(\xi_3^C = j),$$

where the random variables ξ_2 and ξ_3 are defined on the probability spaces $(\Omega, \Sigma, P_{A_C})$ and $(\Omega, \Sigma, P_{B_C})$ respectively, $\xi_1(\omega) = \xi_2^C(\omega) = \xi_3^C(\omega)$, $\omega \in \Omega$. By means of f_2^C, f_3^C, f_2 and f_3, we can also determine the shifts between the duplicates in chronologically incorrect lists. However, those determined by the system of chapter duplicates in C can be found from f_2^C and f_3^C with the help of the machinery described above, whereas the duplicates themselves may not belong to C. Investigating f_2^C or f_3^C for different C, we can study the shifts' structure in more detail. Certain examples of f_2^C for the Popes' list (their nationalities) are shown in Fig. 8.

11 The card deck problem

We now turn to the *card deck problem*. We call two parts of the final deck K *duplicates*, if they contain cards numbered identically or similarly before the original deck is shuffled.