Denote the relation name mean with respect to the permutations by

$$\alpha = Ml(u_i, u_j) = Ml_0(u_i, u_j)/c(k_i, k_j) \tag{3}$$

for any pair (i, j), except for the case where $i = j$ and u_i is a unique name in the list (we do not consider such pairs). We also assume with respect to X that the multiplicity of any name in it is much less than its length $|X| = N$.

Fix the length p, $p \ll N$, of the relating neighbourhood. We may then calculate that, with the said assumptions, the mean non-normed relation $l_0(u_i, u_j)$ of the pair of names u_i and u_j with multiplicities k_i and k_j, respectively, is proportional to

$$c(k_i, k_j) = \begin{cases} 2k_i \times k_j, & i \neq j, \\ k_i(k_i - 1), & i = j. \end{cases} \tag{4}$$

By definition, we put $c(a_r, a_s) = c(k_i, k_j)$, $u(a_r) = u_i$, $u(a_s) = u_j$, for $a_r, a_s \in X$. Here, we discuss the calculation of the mean $Ml_0(u_i, u_j)$ for the case $i \neq j$.

We can represent the scheme of equally likely permutations of names in X as the result of the consecutive placing of N names in N positions in the list, each name occupying one of the remaining vacant places with the same probability. Meanwhile, their turn to be placed can be chosen arbitrarily but must be fixed a'priori. We will assume that,before placing k_j of copies of a name u_j, all k_i of the copies of u_i have already been placed. By assumption, k_i, k_j, $p \ll N$; therefore, we will neglect the number of cases where two copies of u_i turned out to be nearby at a distance of less than p in the list X, compared with the total number of methods of placing k_i. We now represent the placing of k_j of the copies of u_j as a Bernoulli test sequence, and regard the trial as a success if it gets into a relating neighbourhood along with one of the copies of u_i already placed. Then $l_0(u_i, u_j)$ equals the number of successes, and the probability of a success in one trial is proportional to k_i, whereas the number of trials equals k_j; therefore the mean number of successes is proportional to $k_i \times k_j$. The case $i = j$ is considered similarly. Thus, the mean $Ml(u_i, u_j)$ does not depend on the pair (u_i, u_j) except if $u_i = u_j$ is a unique name in the list.

12. Relation measure. The problem of separation of strong and weak relations in a chronicle. In accordance with the assumptions of Item 10 regarding the duplicate appearance mechanism, we introduce a relation measure for two determining neighbourhoods $\Delta_r(k)$ and $\Delta_s(k)$, $k < r \leqslant s < N - k$, in the list X, viz.,

$$L_0(\Delta_r(k), \Delta_s(k)) = \frac{c}{(2k+1)^2} \sum_{i=r-k}^{r+k} \sum_{j=s-k}^{s+k} l(a_i, a_j), \tag{5}$$

where c is a certain convenient constant.

Definition 7. The number $L_0(\Delta_r(k), \Delta_s(k))$ is called the *relation of two neighbourhoods* $\Delta_r(k)$ and $\Delta_s(k)$ in the list X.

If X contains no duplicates, and the assumptions of Item 11 are valid, then as seen from (5), the mean value of the relation $L_0(\Delta_r, \Delta_s)$ does not depend on Δ_r and Δ_s, and equals $c \cdot \alpha$, where c and α were defined in (3) and (4). Here, we imply the mean with respect to the permutations where r and s are fixed.