duplicates, the newly calculated matrix $K\{t\}$ ideally satisfies the frequency-damping principle both with respect to the rows and the columns.

The agreement of these results with the duplicates of the system marked on the GCD is manifest (Fig. 65), e.g., for the name "John". The investigation of the enquête-codes of the principal heroes of the Trojan and Gothic (6th c. A.D.) wars, wars with the Tarquins (exile in the 6th c. B.C.) and in Italy in the 13th c. A.D., duplicates of the T series in Fig. 65, shows that they have the 13th-c. war as their "original". In Italy, John is one of the principal characters in the history of the T-series wars. We construct the graph by marking years from the 1st A.D. to the 17th c. A.D. off on the horizontal, and the frequency of the name "John" in the dynastic stream of popes on the vertical axis (see Fig. 68). It is explicitly seen that they concentrate around the mid-6th c. A.D., the end of the 7th c. A.D., 10th c. A.D. and the end of the 13th c. A.D. In other words, the concentration of "Johns" on the time axis is at the duplicates of the series T, denoted on the GCD and Figs. 65, 66 by black triangles. A duplicate of T, placed at the end of the 7th c. A.D., is localized in the Byzantine Empire, whose history is also subjected to "convolution". This is the time of the well-known crisis and war in Byzantine history, Justinian II (duplicate of Justinian I from the 6th c. A.D.). The duplicates of series T in Byzantine history are sometimes different from their corresponding ones in the history of Rome by *c.* 100 years. In our case, Justinian I from the 6th c. and Justinian II from the end of the 7th c. A.D. are unique Justinians in the history of the Byzantine Empire.

A similar method was applied to the same list of popes, but with their nationalities taken as "names" (the data taken from the traditional tables [74], [119]). As in the case of the name investigation, a rectangular matrix of 51 rows (according to the number of nationalities) and 170 columns (according to the number of decades) was made by A. Makarov. The nationalities were ordered as they appeared in the popes' list. We also include antipopes and gaps as two "names" in order to see the evolution of these two periods in the history of the papacy, too. We then constructed a square matrix of order 170×170 from the latter rectangular. Though satisfying the frequency-damping principle "to the first approximation" (i.e., the graph of $K_{\mathrm{aver}}(t)$ possesses one absolute and explicit maximum, and then more or less vanishes), the calculation of the graphs of $K_{\mathrm{aver}}^{ij}(t)$ and $K_{\mathrm{aver}}^{pq}(t)$ (see their definition above) showed that the list under investigation did contain duplicates. It is remarkable that the duplicates are associated with the same two basic shifts by *c.* 333 and 1,053 years and their difference of 720 years. Thus, the nationalities first appearing in 620–630 A.D. completely vanish as soon as in two decades, and then again appear in 1380–1420 A.D. The difference between these two splashes is *c.* 750–760 years, which is quite close to the 720-year shift. The nationalities which first appear in 280–290 A.D. then vanish after 320 A.D., and are again "reborn" in 640–650 A.D. as the only local splash in 320–340 years. These two splashes are unique in the whole of the matrix row. Thus, we have here an explicit expression of the shift by *c.* 333 years. Finally, the nationalities first appearing in 50–150 A.D. again reappear (after *c.* 1,050 years) in 1080–1210 A.D., which is, obviously, due to the 1,053-year shift. No other shifts were discovered in investigating the nationality matrix. The method for analyzing the graphs of $K_{\mathrm{aver}}^{ij}(t)$ was also applied to the name matrix constructed by A. Makarov for the list of Byzantine patriarchs (pontifices), beginning with 317 A.D.