CHAPTER 8

Tilt angle between the ecliptic and the equator in the Almagest

1. PTOLEMY'S CONCEPT OF THE ECLIPTIC TILT ANGLE VALUE AND SYSTEMATIC ERROR γ

Tilt angle $\varepsilon(t)$ between the ecliptic and the equator is one of the most important values in astronomy. It is necessary to know this angle in order to estimate the ecliptic coordinates of the stars, regardless of the exact method used for said estimation. One can use the astrolabe, as the Almagest text suggests, or use special cosmospheres for conversion from equatorial coordinates, as it was done in the Middle Ages. Other methods counld also have been used, qv in Chapter 2 and the Introduction. It is presently known that the angle of $\varepsilon(t)$ varies over the course of time according to the following rule:

$$\varepsilon(t) = 23°27'8.2849'' + 46.8093''t + 0.0059''t^2 - 0.00183''t^3,$$

where t stands for time counted in centuries backwards from 1900 A.D. (see formula 1.5.3).

The text of the Almagest contains detailed descriptions of how angle ε should be measured, and also the actual instruments that were used for this purpose, qv in Chapter I.12 of the Almagest ([1358]). It is claimed that these measurements resulted in the calculation of the 2ε value that equalled 11/83 of a full circle, or, in modern terms, $\varepsilon_A = 23°51'20''$. Here the value of ε_A stands for the value of angle ε known to the author of the Almagest.

When the author of the Almagest was compiling the star catalogue, he must have used a known value of angle ε, recording it with his instrument (astrolabe, cosmosphere etc). The error in the estimation of the real ε value made by the author of the catalogue would result in the entire celestial sphere as a whole shifted by a certain angle equal to the rate of this error. In other words, the error made in the representation of angle ε on the astronomical instrument leads to a systematic error inherent in the coordinates of all the stars in the catalogue – or, more specifically, the part of the catalogue that was measured with this instrument. It is easy enough to understand that a systematic error of this sort would primarily affect the latitudes of the stars. It is this very systematic error that we educed in Chapter 6 when we were trying to calculate $\gamma_{stat}(t)$ for different values of t. The temporal dependency of the error is primarily defined by the true value of angle $\varepsilon(t)$ changing gradually over the course of time. This alteration is uniform and virtually linear within the confines of the a priori chosen time interval $0 \le t \le 25$.

When the author of the Almagest star catalogue made a mistake in the determination and the fixation of angle ε with his instrument, he altered the value of ε, making it either greater or smaller than the real value; the catalogue would thus either gain or lose age according to the tilt of the ecliptic to the equator. Any of these possibilities could become a reality with the probability of 0.5. What we observe de facto is a manifestation of the most likely option, namely, the value of ε as represented by the Almagest catalogue equals the real value of $\varepsilon(t)$ for the approximate epoch of 1200 B.C., qv in Chapter 6. The compiler of the Almagest had thus made the star catalogue a great deal older.

Let us assume that the Almagest catalogue was compiled in time moment t and that its author considered the tilt angle between the ecliptic and the equator to equal 23°51'20", which is the value stated in the Almagest. Let us also assume that the compiler of the catalogue tried to fix this value of the angle on his astronomical instrument designed for the estimation (via direct observation or re-calculation) of ecliptic stellar coordinates. If we are to consider that the observer's error value lies within the allowed range $\pm\Delta\varepsilon$ defined by the instrument manufacture precision, the summary error of angle ε as fixed by the instrument would equal

$$\varepsilon_A - \varepsilon(t) \pm \Delta\varepsilon = 23°51'20'' - \varepsilon(t) \pm \Delta(\varepsilon).$$

Let us compare the value of this error with the confidence strip $\gamma_{stat}(t) \pm \Delta\gamma$ of systematic error γ as well as the set of γ for which it is possible to superimpose the stellar configuration of the Almagest's informative kernel with the corresponding calculated stellar configuration, and with guaranteed latitudinal precision rate equalling 10', qv in Chapter 7, which also tells us that the last set is non-empty for all intervals but $6 \leq t \leq 13$. Let us choose the values estimated by celestial area *Zod A* for $\gamma_{stat}(t)$ since, as it has been stated above, Almagest catalogue part *Zod A* possesses a single systematic error γ. The confidence strip of γ is more narrow for this part of the catalogue; furthermore, all the stars of the informative kernel are either located in *Zod A* or its immediate vicinity, qv in Chapter 7.

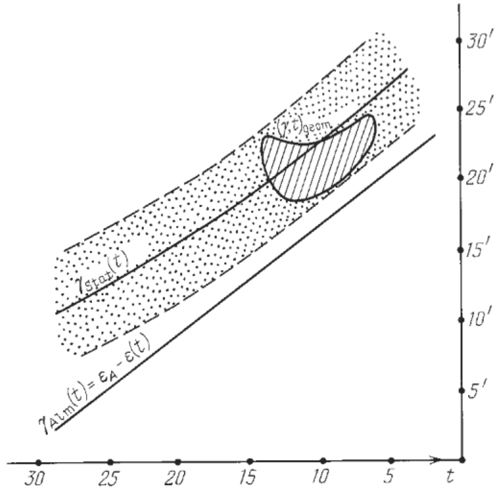

Fig. 8.1. Confidence strip $\gamma_{stat}(t) \pm \Delta\gamma$ estimated for *Zod A*: the set of possible $\gamma_{geom}(t)$ values for the geometrical dating procedure, as well as the dependency graph for the deviation $\varepsilon = \varepsilon_A$, as indicated in the Almagest and the true value of this angle.

In fig. 8.1 we see the confidence strip $\gamma_{stat}(t) \pm \Delta\gamma$ estimated by celestial area *Zod A* with a confidence level of 0.998. We also see the set of acceptable $\gamma_{geom}(t)$ geometrical dating procedure values for which the maximal latitudinal discrepancy of the Almagest informative kernel stars does not exceed 10', qv in Chapter 7. Finally, in fig. 8.1 we see a dependency graph for the aberration of $\varepsilon = \varepsilon_A$ as given in the Almagest from the real value of this angle: $\gamma_{Alm}(t) = \varepsilon_A - \varepsilon(t)$.

	Arc length (ring length) in millimetres depending on the ring radius in metres		
Arc	*0.5 (3.14)*	*0.75 (4.71)*	*1.0 (6.28)*
2'30"	0.4	0.5	0.7
5'	0.7	1.1	1.4
10'	1.5	2.2	2.9
1°	8.7	13.0	17.5

Table 8.1. Arc lengths of 2.5', 5', 10' and 1° in millimetres as indicated on the rings whose radius equals 50 cm, 75 cm and 1 m.

Fig. 8.1 demonstrates that the graph of $\gamma_{Alm}(t)$ to be in close propinquity with the "geometrically valid" area $(\gamma, t)_{geom}$ and the confidence strip that surrounds $\gamma_{stat}(t)$, albeit not crossing it – the latter would take place if we transposed the graph of $\gamma_{Alm}(t)$ upwards by circa 2.5'. Then it shall automatically begin to cross both the confidence strip and the "geometrically valid" area shifted towards the respective edge of the confidence strip. A shift of 6.5' upwards shall make the graph of $\gamma_{Alm}(t)$ virtually coincide with the graph of $\gamma_{stat}(t)$, while still crossing the "geometrically valid" area. The value of the shift needed for this purpose corresponds to the allowed variation of $\Delta\varepsilon$ with ε_A fixed on the instrument and gives us an idea of just how precise the manufacturers of the astronomical instrument could have been. Table 8.1 contains the arc length values of 2.5', 5', 10' and 1° (in mm) on an astronomical instrument (astrolabe, cosmosphere etc) with a radius of 50 cm, 75 cm and 1 m.

From table 8.1 we can see that for the ε angle fixation error $\Delta\varepsilon$ of an astronomical instrument, the value of 2.5'-5' is very real for the Middle Ages. It corresponds to the linear size fluctuation range of a mere 0.5-1 mm.

Thus, the ecliptic tilt values that we have discovered in the Almagest catalogue correspond with the value of ε_A contained in the text of the Almagest.

2. THE PETERS ZODIAC AND THE SINE CURVE OF PETERS

Paragraph 1. The book of Peters and Knobel ([1339]) contains an important discrepancy graph that Peters obtained from his analysis of the Almagest catalogue. The sine curve of this graph shall be referred to as the "latitudinal sine curve of Peters" (see [1339], page 6). This curve indicates the present of certain systematic errors in the Almagest.

In the present section we shall explain why the "sine curve of Peters" is inherent in the Almagest catalogue.

Paragraph 2. Let us consider the location of the ecliptic Π for $t = 18$, or 100 A.D. We shall mark the vernal equinox point $Q(18)$ upon it. We shall proceed to divide the ecliptic into 360 degrees, using the vernal equinox point for initial reference, qv in fig. 8.2.

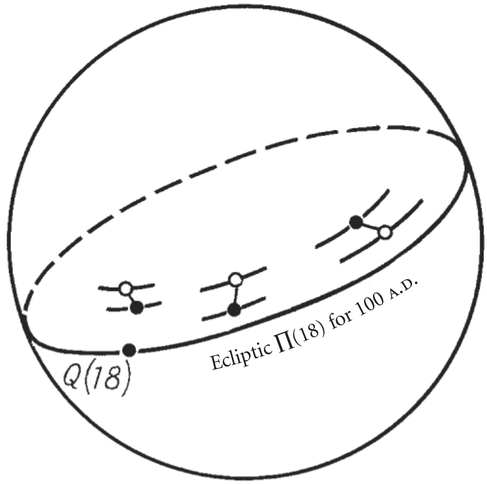

Fig. 8.2. Position comparison for real stars in 100 A.D. and their positions as indicated in the Almagest.

Now let us mark the positions of the real stars for 100 A.D. as black dots on the celestial sphere, and the positions of the same stars in the Almagest as white dots. Respective dot pairs (black and white) are linked together with segments in fig. 8.2, so as to make the correspondences clear.

We can calculate the latitudinal difference for each such pair, or the latitudinal discrepancy in other words. We are thus calculating the difference between the latitude of star i in the Almagest and the real latitude of this star for 100 A.D. Peters studies the Zodiacal stars of the Almagest from this position in [1339]. However, he appears to have missed some of them. The Almagest contains a total of 350 Zodiacal stars. As we point out in [1339], page 17, Peters only chose 218 stars for his study of the Zodiacal star longitudes, without specifying the selection principles. The exact amount of stars studied by Peters in his research of he latitudes isn't given anywhere in [1339], but one can assume him to have take the same stars as he did in his research of the longitudes.

Let us calculate latitudinal discrepancies for all the stars from the zodiacal list and represent them on the graph. This shall require taking the longitude of the stars and marking it on the horizontal axis, and then presenting the value of the latitudinal discrepancy on the vertical. This shall result in a certain agglomeration of points drawn on the plane which we

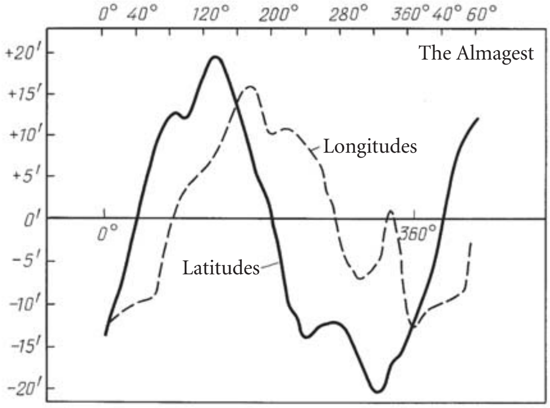

Fig. 8.3. The smoothing curves of Peters for 100 A.D. (latitudinal and longitudinal).

shall be referring to as the "error field". Once we divide the longitudinal scale into 10-degree segments and average each one of those, we can build the smoothing curve as seen in fig. 8.3. This curve, in turn, can be approximated by the optimal sine curve according to the minimal criterion of the mean-square discrepancy.

A similar procedure can be performed for the longitudes. We shall come up with another smoothing curve as a result which is represented in fig. 8.3 as a dotted curve. We shall talk about this curve later.

Let us find a natural explanation of these curves.

Paragraph 3. Let us begin with a study of the latitudinal sine curve of Peters. We must note that the natural mechanism that allows us to explain the inclusion of systematic errors into the latitudes of Zodiacal stars. This is the error in the location of the observer's ecliptic plane as compared to that of the real ecliptic for the moment of observation which isn't known to us a priori.

Let us return to our consideration of ecliptic $\Pi(t_0)$ for observation moment t_0. Equinox point $Q(t_0)$ is marked in fig. 8.4 as the beginning of coordinates. Above we see the latitudinal error field for $t = 18$. Let us do the same for the Almagest catalogue star observation moment t_0 and draw the corresponding latitudinal error field in fig. 8.5. The smoothing curve shall be marked $c(X, K(t_0, 0, 0)$ – see the dotted curve in fig. 8.5. Let us explain this indication. As above, X is used for referring to the Almagest catalogue. $K(t, \beta, \gamma)$ is used for referring to the real catalogue $K(t)$ referring to real star positions for epoch t perturbed by parameters β and γ, qv in Chapter 6. Thus, $K(t_0, 0, 0)$ is a catalogue which was not subject to random perturbation that shows real star positions for observation moment t_0 that we do not know a priori.

We already explained it in Chapter 6 that in order to find the optimal ecliptic rotation in the square average sense, we have to solve the correspondent regression problem. For this end we shall have to use a two-parameter sinusoidal family as the family of ap-

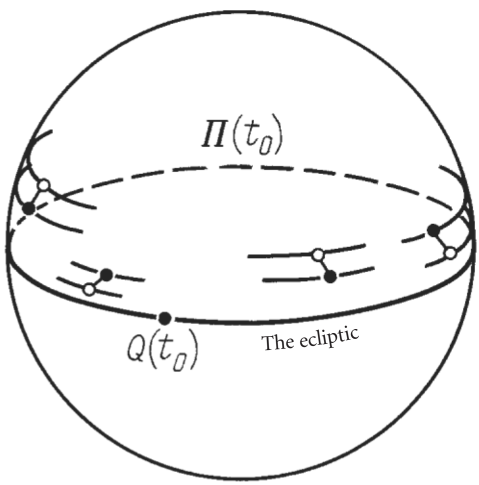

Fig. 8.4. The system of accounting for latitudinal errors.

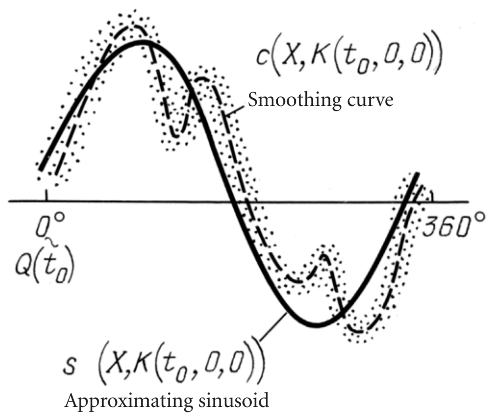

Fig. 8.5. The dotted curve indicates the smoothing curve $c(X, K(t_0, 0, 0)$. The continuous curve is the approximating sinusoid $s(X, K(t_0, 0, 0)$.

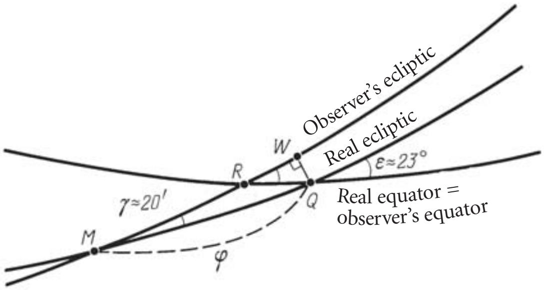

Fig. 8.6. The observer's ecliptic, the real ecliptic and the real equator.

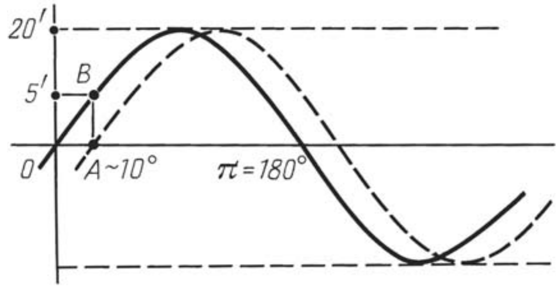

Fig. 8.7. Alteration of the sinusoid phase.

proximating curves. The first parameter of this family shall be defined by the amplitude of a sine curve, and the second – by its phase. We solved this problem in Chapter 6 – for the Almagest in general as well as its different parts in particular; among the latter – the Zodiac which is of interest to us at the moment. Let us define the optimal approximating sine curve as $s(X, K(t, 0, 0))$ – see the continuous curve in fig. 8.5. The parameters of the sine curve shall be defined as A^* (amplitude) and φ^* (phase).

Paragraph 4. It would be a good idea to discuss the concept of approximating sine curve phase. The matter is that the phase is estimated with the precision rate of 15 degrees at best. Let us provide two virtually equivalent explanations to this fact. The first is based on the analysis of how the observer's error in the estimation of the ecliptic plane affects the phase of the approximating sine curve. One sees the following objects in fig. 8.6. Firstly, it is the real equator for observation moment t_0. This equator, as we have explained above, can be considered all but identical with the observer's equator. Secondly, it is the real ecliptic for moment t_0 and the observer's ecliptic.

We know the angle between the observer's ecliptic and the real ecliptic to approximately equal 20', which is the observer's error γ. The angle between the equator and the ecliptic equals ε, or circa 23°. It doesn't matter which one of the ecliptics we are considering at the moment since the angle between them is minute as compared to 23°. The arc in fig. 8.6 represents the observer's error in estimating the vernal equinox point. As we already know, this error is roughly equivalent to 10', which is the scale grading value of the Almagest catalogue. Let us assume arc RQ is approximately equal to 10'; in this case arc distance WQ shall be approximately equal to 10' × sin20°, or roughly 5'. In this case, arc distance φ, or arc MQ from fig. 8.6, shall be equal to circa 5' / sin20', or around 15°. All we have to point out us that arc MQ gives us a precise representation of the approximating sine curve phase. We are counting the sine curve phase starting with the vernal equinox point $Q(t)$ upon the real ecliptic $\prod(t)$.

Thus, several-minute perturbations in observer ecliptic estimation perturb the sine curve phase by a factor of several degrees, making the phase "unstable".

The very same phenomenon receives an explanation if regarded as part of the smoothing curve $c(X, K(t, 0, 0))$ approximation problem with the optimal sine curve of $s(X, K(t, 0, 0))$.

Approximating the smoothing curve by the optimal sine curve we reach the minimal value of the possible square average error. One has to allow for a certain variation of this minimum due to the fact that the optimal sine curve's parameters in general fail to concur precisely with the actual observation error. Allowing for 5-minute variations of the square average aberration minimum we must note that a 10-degree phase variation of a sine curve with the amplitude of 20' changes the ordinate of any sine curve point by a maximum of 5'. For a standard sine curve with an amplitude of 1 and a phase of 0 drawn as an continuous curve in fig. 8.7 segment OA shall be approximately equal to arc OB, since we are presently considering segment OA comparatively small, or equalling 1/6 radians (10 degrees). In this case segment AB comprises 1/6 of the amplitude, or approximately 3.3'. Therefore, a three-minute perturbation

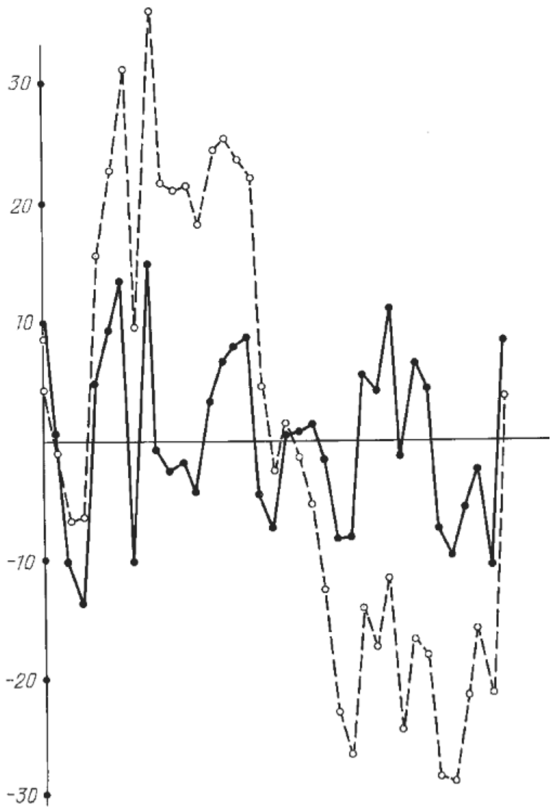

Fig. 8.8. The Almagest, $t = 9$. The dotted curve represents the initial Peters sinusoid, and the continuous curve stands for the same after the subtraction of the systematic error value.

of the square average discrepancy can result in a ten-degree phase alteration of the approximating optimal sine curve.

Paragraph 5. In the preceding chapters we already estimated the possible dating interval of the Almagest catalogue, namely, we discovered that t_0 lies on the interval between 6 and 13, or approximately 600 A.D. and 1300 A.D. Therefore it would be particularly interesting if we studied the approximating sine curves $s(X, K(t, 0, 0))$ for this very interval of possible datings. It turns out that they don't alter too much inside the interval between 600 A.D. and 1300 A.D., or prove to be poorly-dependent on t_0. More precisely, the maximal amplitude of A^* changes from 26' for $t_0 = 6$ to 20' for $t_0 = 13$ inside the interval between 600 A.D. and 1300 A.D. The corresponding phase shifts of φ^* take place between the values of –17° and –18° in relation to the corresponding equinox point $Q(t_0)$ on the ecliptic $\prod(t_0)$. We can therefore regard any smoothing curve $c(X, K(t_0, 0, 0))$ as a "typical representative" of the class, where t_0 can assume any value from 6 to 13. It would be natural to consider the middle of the time interval, namely, the value of $t_0 = 9$.

Let us demonstrate how the smoothing curve $c(X, K(t_0, 0, 0))$ looks at $t_0 = 9$ before and after the optimal sine curve subtraction, or, in other words, before and after the exclusion of the systematic errors that we discovered. In fig. 8.8 one sees that the smoothing curve $c(X, K(t_0, 0, 0))$ is close to a sine curve for $t_0 = 9$. The parameters of the optimal sine curve for $t_0 = 9$ are as follows: the amplitude equals to 24', and the phase to –17°. The smoothing curve is drawn as a dotted curve in fig. 8.8. Excluding observer's ecliptic estimation error from catalogue X is equivalent to subtracting the optimal sine curves with the parameters being $A^* = 24'$ and $\varphi^* = -17°$ for $t_0 = 9$. As a result, the latitudinal discrepancy smoothing curve assumes the form drawn as a continuous curve in fig. 8.8. One can clearly see the difference between the dotted curve and the continuous curve; the latter fluctuates around the abscissa axis and corresponds to the zero average error of the observer in the estimation of the ecliptic position. It is obvious that the error field is now approximated by a degenerate sine curve, or a mere straight line that becomes superimposed over the abscissa.

Conclusion. The compensation of observer errors on the discovered possible dating interval of the Almagest catalogue, namely, 600-1300 A.D., results in the disappearance of such effects as the latitudinal sine curves of Peters.

Paragraph 6. Let us return to the sine curve of Peters in the latitudes of the Almagest catalogue. Since it is possible that Peters did not account for all of the Zodiacal stars in his calculations, we have re-calculated and built a graph similar to that of Peters for $t = 18$, or 100 A.D., qv in fig. 8.3. We have considered all the Zodiacal stars of the Almagest except for several rejects with a latitudinal discrepancy of more than 1.5°. The data were taken from [1339]. We processed nearly all of 350 Zodiacal Almagest stars.

The result of our calculations can be seen in figs. 8.9 and 8.10 together with the latitudinal error field of the Almagest Zodiac for $t = 18$. This field consists

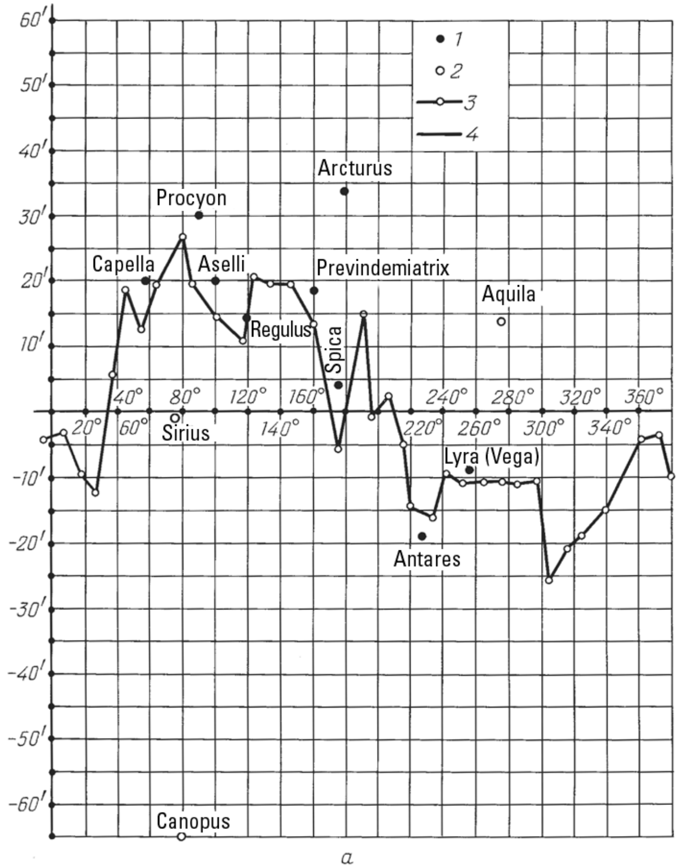

Fig. 8.9. The Peters curve that we calculated for the Almagest zodiac, $t = 18$.

of 350 points scattered across a plane. The continuous zigzag represents the smoothing curve $c(X, K(t_0, 0, 0))$. It is plainly visible that it bears qualitative semblance to the curve of Peters in fig. 8.3. In general, the behaviour of our adjusted curve in fig. 8.9 is similar to that of the Peters curve in fig. 8.3. However, there are some minor differences which are apparently explained by Zodiacal star selection principle used by Peters which remains unknown to us.

In fig. 8.10 one also sees the optimal sine curve $s(X, K(18, 0, 0))$. Its parameters are as follows: an amplitude of 16' and a phase of –22°, qv in Chapter 6.

Paragraph 7. Above we have considered various properties of the latitudinal error field as related to the real observation moment t_0. Let us now examine the same field for the arbitrary moment t which does not coincide with t_0. We see the following in fig. 8.11:

1) The real ecliptic $\prod(t)$ for observation moment t_0.

2) Observer ecliptic represented by a dotted curve and not equal to $\prod(t_0)$ due to the effects of the observation error made by the Almagest catalogue compiler.

3) The real ecliptic $\prod(t)$ for any other fixed moment t.

Vernal equinox points $Q(t_0)$ and $Q(t)$ are drawn on the ecliptics $\prod(t_0)$ and $\prod(t)$. Point N corresponds to the crossing of said ecliptics. The distance between point M and ecliptic $\prod(t)$ is rather small, that is to say, it doesn't exceed 20' if $|t - t_0|$ does not exceed 2000 years. Therefore, the latitudinal error field as related to ecliptic $\prod(t)$ should be approximated as a sum of two sine curves. The first results from observation error made in time moment t_0 and was discussed in detail above. The phase of this sine curve

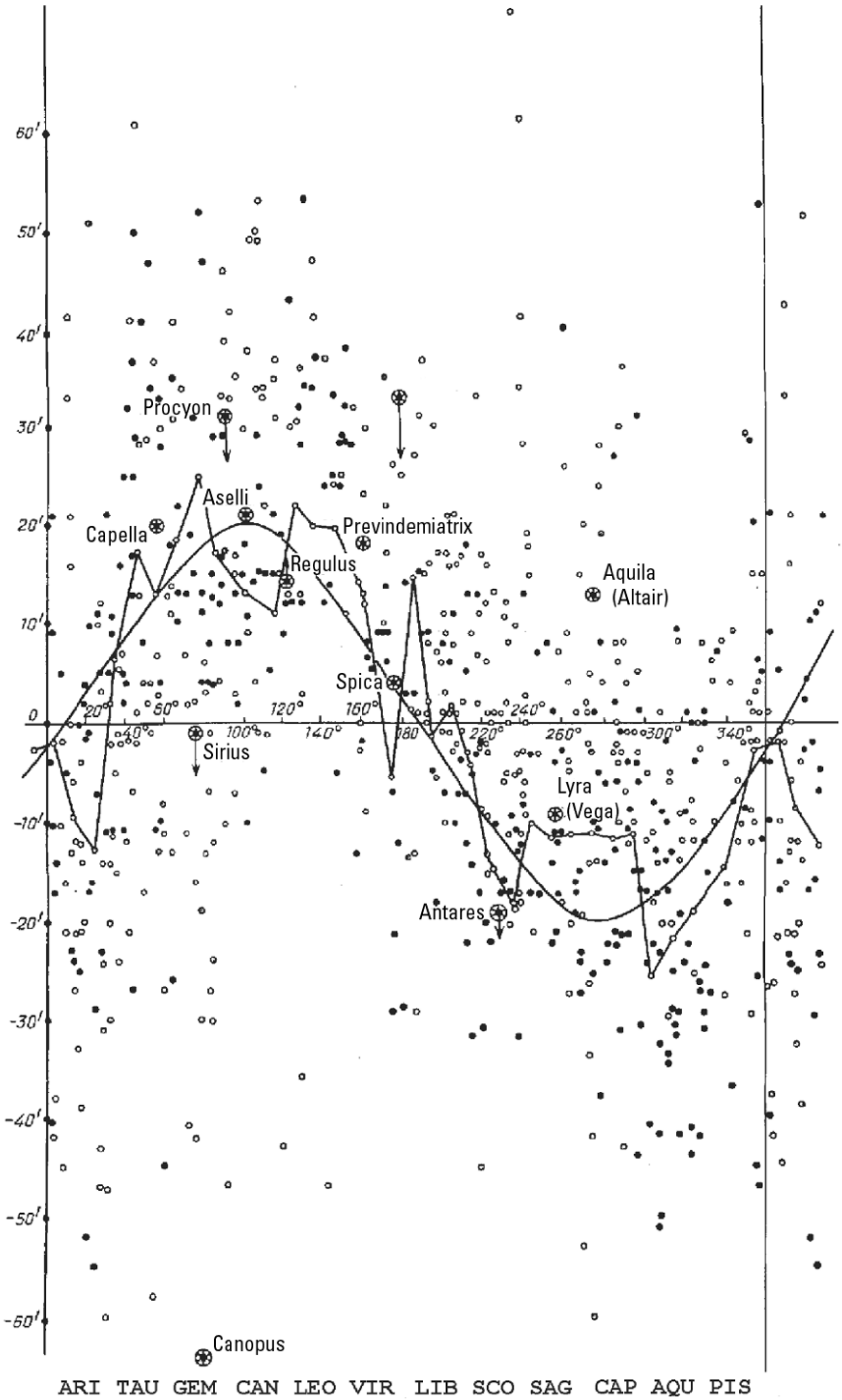

Fig. 8.10. Error field for the Almagest zodiac, $t = 18$. Zodiacal stars are represented by black dots, others – by light ones. The zigzag is the Peters curve approximated by 10-degree intervals, whereas the smooth curve stands for the optimal sinusoid.

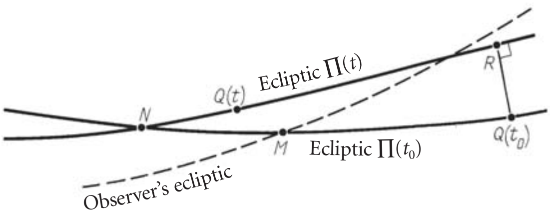

Fig. 8.11. The real ecliptic for the moment of observation, the observer's ecliptic and the position of the real ecliptic for a different moment in time.

as related to vernal equinox point $Q(t)$ on ecliptic $\prod(t)$ approximately equals the sum of its phase as related to vernal equinox point $Q(t_0)$ – see arc $MQ(t_0)$ in fig 8.11, and the arc distance $RQ(t)$. We are referring to an algebraic sum here, or a sum with either a positive or a negative value. The arc $RQ(t)$ equals the precession value for the time $t - t_0$.

The second sine curve $s_{t,\,t_0}$ represented in fig. 8.12 as a continuous curve, results from the discrepancy between ecliptic $\prod(t)$ and ecliptic $\prod(t_0)$. It has an approximate amplitude of 47" / $|t - t_0|$, qv in [1222] or in Chapter 1. Its phase is estimated by precession formulae from section 5 of Chapter 1 which were taken from [1222] originally.

The resultant approximating curve represents the sum of these two sine curves. This curve has a single local maximum and a single local minimum upon the circumference, or the ecliptic.

This implies the following simple statement. Let us regard the two time moments of t_0 and t. Then the smoothing curve $c(X, K(t_0, 0, 0))$ shall approximately coincide with the sum of the two curves $c(X, K(t, 0, 0)) \approx c(X, K(t_0, 0, 0)) + s_{t,\,t_0}$. Thus we can claim that a sine curve like that of Peters for time moment t approximates the sum of a similar sine curve for moment t_0 and the one corresponding to the rotation of the ecliptic over the time $t - t_0$ (between t_0 and t). This is a general statement valid for all couples of t and t_0.

Paragraph 8. Now let us consider the resultant approximating curve for 100 A.D., or $t = 18$. We have just explained that one needs to add up two sine curves for this purpose. The first corresponds to the real observation moment t_0, and the second to time moment t for which the resultant approximating curve is calculated. Let us choose $t_0 = 9$ as the "real observation time", or roughly 1000 A.D. This value of t_0 is the middle of the possible Almagest catalogue dating interval between 600 and 1300 A.D., or $t = 13$ and $t = 6$, that we have discovered. The first sine curve (see the dotted curve in fig 8.13) has the amplitude of 24' and the phase of –5°, which is a sum of –17° (see arc $MQ(t_0)$ in fig. 8.11), and 12°, of the precession for some 900 years.

The second sine curve (see the fine continuous curve in fig. 8.13) corresponds to the choice of the moment $t = 18$, or 100 A.D., qv above. Its amplitude roughly equals 47" × 9 ≈ 7', qv above, and its phase approximates 160°, qv in Chapter 1. On the fragment between –20° and 160° as seen in fig. 8.13 this curve is located under the abscissa, or has a negative value. Adding up the two sine curves we shall get the resultant approximating curve drawn as a bold continuous curve in fig. 8.13.

Thus, the latitudinal discrepancy sine curve dis-

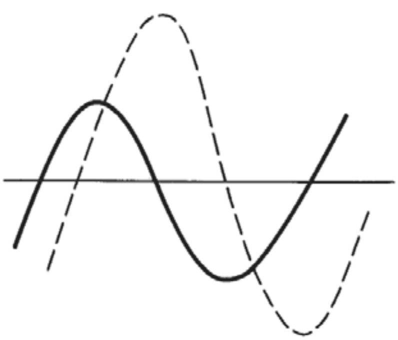

Fig. 8.12. A pair of sine curves whose sum roughly defines the latitudinal error field.

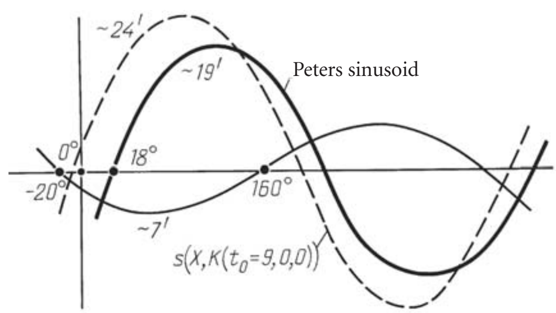

Fig. 8.13. A sum of two sinusoids yields a Peters sinusoid (bold curve).

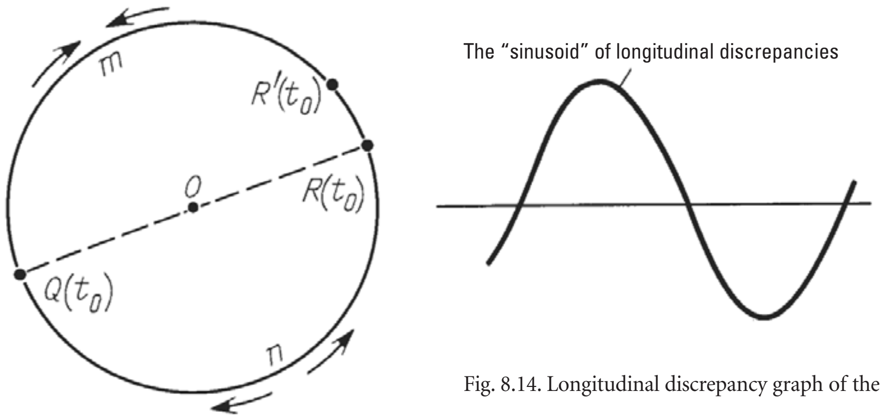

Fig. 8.14. Longitudinal discrepancy graph of the zodiacal stars.

covered by Peters under the assumption that the Almagest catalogue was compiled in 100 A.D. is a sum of two sine curves, namely, the observation moment sine curve resulting from the incorrect estimation of the ecliptic position by the observer, and the sine curve that results from the angle between the ecliptic of 100 A.D. and the ecliptic of the observation time.

Paragraph 9. Let us conclude with turning to the longitudinal sine curve of Peters (see the dotted curve in fig. 8.3). The mechanism described above explains the genesis of the latitudinal sine curve; however, it hardly affects the longitudes of the Zodiacal stars. Therefore, the incorrect estimation of the ecliptic by the observer does not result in a manifest longitudinal sine curve. Nevertheless, we can witness a weakly-manifest sine curve to appear in longitudes as well. Let us assume that the mediaeval observer made an error in his estimation of the vernal and autumnal equinox points, or, which is virtually the same, measured the coordinates of the basis stars with insufficient precision. Bear in mind that unlike the latitudes that were always counted from the ecliptic ring of the astronomical instrument, fixed in its construction with a permanent error, stellar longitudes were counted off several bright basis stars. Otherwise one would have to measure angles larger than 180°, which is an arduous procedure (see Chapters VII.3 and VII.4 of the Almagest ([1358])). This circumstance is illustrated in fig. 8.14.

Lack of precision in the estimation of the equinox points by the observer shall lead to a de facto division of the ecliptic into two unequal parts by the points $Q(t_0)$ and $R'(t_0)$. Here $R'(t_0)$ stands for the erroneous position of the autumn equinox point and $R(t_0)$ being the real autumn equinox point. The length of arc RR' may be rather small, around 10' – 15', remaining within the precision threshold of the Almagest. Some of the Zodiacal longitudes could be measured from the vernal equinox point Q, or a certain group of basis stars, whereas other longitudes would be measured from the autumn equinox point R, or from another group of basis stars. As a result, stellar longitudes on segment QmR' shall be "compressed" by roughly 15', whereas on segment QnR' they shall, on the contrary, be expanded by roughly 15'. Therefore, calculating the longitudinal discrepancy graph of the Zodiacal stars, we shall end up with a sinusoidal curve, qv in fig. 8.14. Bear in mind the relatively small value of the 10' – 15' error, which is the amplitude of the longitudinal Peters sine curve as seen in fig. 8.3.