CHAPTER 7

The dating of the Almagest star catalogue. Statistical and geometrical methods

1. THE CATALOGUE'S INFORMATIVE KERNEL CONSISTS OF THE WELL-MEASURED NAMED STARS

The analysis of the Almagest star catalogue related in Chapters 2-6 had the objective of reducing latitudinal discrepancies in star coordinates by compensating the systematic error as discovered in the catalogue.

As a result, we have proven that the Almagest compiler's claim about the precision margin of his measurements being less than 10' is justified – insofar as the latitudes of most stars from celestial area *A* are concerned, at least. We believe this circumstance to be of paramount importance.

However, we can only date the Almagest catalogue by considering fast and a priori precisely measurable stars. In other words, dating purposes require individual error estimates. Our statistical characteristics can tell us nothing about the precision of actual star coordinate measurements or the stars measured with the greatest precision.

The choice of such stars can only be defined by reasonable considerations based on known practical methods of measuring stellar coordinates as used by the ancients (see Chapter 1). It is known that the measurements of most stars' coordinates have always been based on the so-called reference stars, whose number is rather small as compared to the total number of the stars in the catalogue.

Let us begin by reiterating a number of considerations voiced in the preceding chapters, which will serve as a foundation of our dating method.

Unfortunately, we do not know which reference star set was used by the author of the Almagest. All we do know is that it must have included Regulus and Spica, since the measurement of their coordinates is discussed in separate dedicated sections of the Almagest. However, it would make sense to assume that the compiler of the catalogue was at his most accurate when he measured the coordinates of named stars. As it was stated above, there are twelve such stars in the Almagest: Arcturus, Regulus, Spica, Previndemiatrix, Cappella, Lyra = Vega, Procyon, Sirius, Antares, Aquila = Altair, Acelli and Canopus.

The identity of Ptolemy's reference stars (as used for planetary coordinate measurements) is an issue that was studied in [1120]. The stars in question turn out to be as follows (Ptolemy actually mentions them as ecliptic reference stars): Aldebaran = α Tau, Regulus, Spica and Antares. Three of them have proper names in the Almagest – namely, Regulus, Spica and Antares. Apparently, Ptolemy also had to add Aldebaran to their number for the purpose of planetary observations. Incidentally, all four stars are included in our table 4.3.

The twelve named stars of the Almagest are bright, clearly visible against their background and providing a convenient basic set of reference points on the celestial sphere. The most important circumstance is that a sufficiently large part of these stars are characterised by substantial proper motion rates, especially Arcturus, Procyon and Sirius.

Seven of the named Almagest stars are located in celestial area *Zod A* or its immediate vicinity. They are as follows: Arcturus, Spica, Procyon, Acelli, Previndemiatrix, Regulus and Antares. Nine of the named stars surround area *A* – the above set needs to be complemented by Lyra = Vega and Cappella. Thus, even if these 12 stars weren't used for reference, their coordinates are still most likely to have been measured with sufficient precision.

However, despite the probable high precision of their coordinates as measured in the Almagest, the stars comprised in this group are by no means of equal importance. Our analysis has revealed the following:

1) Canopus is located far in the south, and measurement precision is greatly affected by refraction in such cases. Therefore, all efforts of the Almagest's compilers notwithstanding, the coordinates of this star as given in the catalogue are a priori known to be more than one degree off the mark.

2) The coordinates of Previndemiatrix as measured by the compiler of the Almagest remain unknown to us – we are only familiar with results of later research ([1339]).

3) Group errors in the environs of Sirius and Aquila fail to concur with the errata inherent in the coordinates of all the other stars, as we have discovered in Chapter 6. We are incapable of calculating the rates of these errors, and, consequently, compensation is a non-option in their case.

Thus, we end up with 8 named stars that we can use for the purpose of dating. The stars that surround them have a single group error in their coordinates – at least, the γ component of this error is the same in each and every case. We shall be referring to these stars as to the *informative kernel* of the Almagest catalogue.

It would make sense to put forth the following hypothesis. If the precision rate claimed by the compiler of the catalogue was actually true, it is guaranteed to manifest as such in the case of the catalogue's informative kernel after the compensation of the group error.

This is the very hypothesis that out method of dating star catalogues relies upon.

However, the fact that the informative kernel of a star catalogue has the ability to assist us with the dating of the latter is far from obvious. In general, the fact that we did manage to reconstruct the true values of random errors inherent in the Almagest catalogue by group error compensation does not imply that the individual errors in the coordinates of the catalogue's kernel stars are the same. It doesn't seem too likely that a discrepancy of this sort actually exists – the central star of a group appears to have the same sort of error in its coordinates as its closest neighbours. However, strictly speaking, the hypothetical existence of such a discrepancy has to be taken into account nonetheless. Apart from that, one mustn't rule out the possibility that the coordinates of a star included in the catalogue's informative kernel were measured with an error margin of more than 10'.

All of the above tells us that if we do manage to find a moment in time conforming to the requirements of our hypothesis, we shall once again prove the correctness of our initial statistical conjectures.

2. PRELIMINARY CONSIDERATIONS IN RE THE DATING OF THE ALMAGEST CATALOGUE BY THE VARIATIONS IN THE COORDINATES OF NAMED STARS

In section 1 we singled out a group of stats that we have called the Almagest's informative kernel. We shall consider its behaviour in detail below. What we shall analyse herein is the behaviour of all 12 named Almagest stars at once. This preliminary study demonstrates perfectly well how much greater the precision rate of the Almagest catalogue becomes after the compensation of its systematic error. It also provides additional explanation to the fact that three named stars out of twelve (Canopus, Sirius and Aquila = Altair) break the homogeneity of the entire sample. We learn

The name of a star and the respective Bailey's number	*Years*					
	1800 A.D.	*1400 A.D.*	*900 A.D.*	*400 A.D.*	*100 A.D.*	*200 B.C.*
Arcturus (110)	37.8	21.2	0.9	19.3	31.4	43.3
Sirius (818)	23.6	18.3	11.7	5.1	1.2	2.6
Aquila = Altair (288)	8.6	9.4	10.5	11.8	12.6	13.4
Previndemiatrix (509)	13	14.3	15.8	17.1	17.8	18.4
Antares (553)	32.6	29.5	25.5	21.6	19.3	17
Aselli (452)	30.5	28.5	25.9	23.2	21.5	19.8
Procyon (848)	11.2	16	21.9	27.6	31.1	34.4
Regulus (469)	17.5	16.6	15.4	14	13	12.1
Spica (510)	2.4	0.7	1.3	3.1	4.2	5.2
Lyra = Vega (149)	15.4	14.2	12.5	10.8	9.8	8.7
Capella (222)	21.9	21.7	21.3	21	20.8	20.6
Canopus (892)	51	54.2	58.2	62.3	64.8	67.3

Table 7.1. Latitudinal discrepancies of the 12 named Almagest stars and their dependency on the presumed dating. The systematic error discovered in the Almagest catalogue isn't compensated herein.

these stars to be "rejects" in relation to all the other named stars. Below in our study of all 12 named stars as a whole we shall be using the coordinates of Previndemiatrix from [1339] which were apparently calculated by Halley. We shall use $\Delta B_i(t, \gamma, \varphi)$ for referring to the difference between the latitude of star i from the informative kernel of the Almagest after the compensation of the systematic error (γ, φ) and the true latitude as calculated for epoch t.

Consideration 1. Let us observe the correlation between the latitudinal precision of the named stars' coordinates in the Almagest with the grade value of the catalogue equalling 10', assuming that the latter contains no global systematic errors. Table 7.1 contains the absolute latitudinal discrepancy values of all 12 named Almagest stars depending on the alleged dating t. In the first column we see the given star's Almagest number (in Bailey's numeration). The rates of latitudinal discrepancies are given in arc minutes.

Table 7.1 demonstrates that for 7 out of 12 named Almagest stars the latitudinal discrepancy exceeds the limit of 10'. The columns that correspond to 100 A.D., which is the Scaligerian dating of the Almagest (Ptolemy's epoch) or 200 B.C. (the epoch of Hipparchus) draw our attention primarily because of the outrageous error in the coordinates of Arcturus – around 30' or 40'. It is peculiar that the brightest and most visible star of the Northern hemisphere would be observed by either Ptolemy or Hipparchus this much worse than all the other stars. Furthermore, the text of the Almagest implies that the coordinates of Regulus were measured several times during the compilation of the catalogue, and that the star in question is known to have been one of the referential points for the measurement of all the other stars in the catalogue. It would be natural to expect that Ptolemy had been exceptionally careful in his measurement of this star; therefore, its latitudinal discrepancy shall be less than 10'. Let us point out that for another bright star on the ecliptic – namely, Spica, whose coordinates had also been measured by Ptolemy during the initial stage to be used for reference later (see Chapter VII.2 of the Almagest, or [1358]), has a latitudinal discrepancy of 5' – less than half the catalogue grade value.

Let us now consider the systematic error that we discovered in the Almagest (see Chapter 6). As the γ compound of this error only varies slightly over the interval between the beginning of the new era and the middle ages, and the variations of the φ value also hardly affect the picture, we shall use the values $\gamma_0 = 21'$, $\varphi_0 = 0$. The value $\gamma_0 = 21'$ is the average value of $\gamma(t)$ for t from the a priori known interval.

The name of a star and the respective Bailey's number	*Years*					
	1800 A.D.	*1400 A.D.*	*900 A.D.*	*400 A.D.*	*100 A.D.*	*200 B.C.*
Arcturus (110)	29.9	15.5	2.3	20	30.5	41
Sirius (818)	44.2	39.2	32.7	25.9	21.8	17.5
Aquila = Altair (288)	27	28.7	30.7	32.5	33.5	34.4
Previndemiatrix (509)	15.6	14.9	13.8	12.6	11.8	11
Antares (553)	13.3	11	8.5	6.2	4.9	3.7
Aselli (452)	13.2	10.2	6.5	2.9	0.9	1.1
Procyon (848)	8.1	4	1.2	6.7	10.1	13.5
Regulus (469)	6.1	3.5	0.4	2.7	5.1	6.2
Spica (510)	5.1	4.9	4.4	3.7	3.3	2.7
Lyra = Vega (149)	5.1	6.7	8.5	10	10.8	11.5
Capella (222)	1.3	1.5	2.1	2.9	3.5	4.2
Canopus (892)	71.5	75	79.2	83.1	85.4	87.6

Table 7.2. Latitudinal discrepancies of the 12 named Almagest stars and their dependency on the presumed dating as given after the compensation of the systematic error in the Almagest stellar coordinates specified by parameters $\gamma_0 = 21'$ and $\varphi_0 = 0$.

We shall proceed with building the table numbered 7.2, which is similar to 7.1, the sole difference being that the systematic error defined by parameters $\gamma_0 = 21'$ and $\varphi_0 = 0$ in all the stellar coordinates is now taken into account and compensated in the calculation of latitudinal discrepancies.

A comparison of the two tables demonstrates the precision characteristics of the named Almagest star coordinates to have improved drastically for all possible datings after the compensation of the systematic error. The latitudes of Regulus and Spica prove to be measured with the precision rate of up to 5' for every alleged dating between the beginning of the new era and the end of the Middle Ages. This correlates well with the fact that these two stars enjoy a great deal of attention in the text of the Almagest – qv in the book itself, Chapter VII.2 ([1358]). Moreover, if we are to place the dating on the interval of $6 \le t \le 10$, or 900-1300 A.D., the latitudinal discrepancy does not exceed 10', or the catalogue scale grade value, for 8 named stars out of 12 – the ones located in celestial area *A* which we discovered in Chapter 6 as we were analyzing the entire stellar aggregate of the Almagest catalogue.

It goes without saying that the above considerations need to be more explicit. In particular, we have to study other values of parameters γ and φ. The present chapter contains extensive calculations and more explicit statements below.

Consideration 2. The following line of argumentation might provide additional information pertinent to the dating of the Almagest catalogue. Let us consider the latitudinal discrepancies $\Delta B_i(t, \gamma, \varphi)$ of a certain Almagest star set *E*, $1 \le i \le n$ as a whole for each moment *t* and all the values of γ and φ. We shall use them for building empirical function graphs of latitudinal error distribution for star set *E*: $F_{t,\gamma,\varphi}(x) = (1/n)\ \#\{i : |\Delta B_i(t, \gamma, \varphi)| \le x\}$, where *n* represents the quantity of stars in set *E*. A comparison of these distribution functions for different values of parameters t, γ and φ can allow us to try finding such a combination of these values that will minimize the latitudinal errors of set *E* stochastically. The error difference rate for different values of t, γ and φ shall be their average difference value. We can obviously come to no quantitative conclusions so far since we only have 12 observations at our disposal, and we shall thus be merely referring to the qualitative picture as a first approximation.

The error difference rate in question can be represented as the area contained between the distribution graphs of the functions $F_{t1,\gamma1,\varphi1}(x)$ and $F_{t2,\gamma2,\varphi2}(x)$

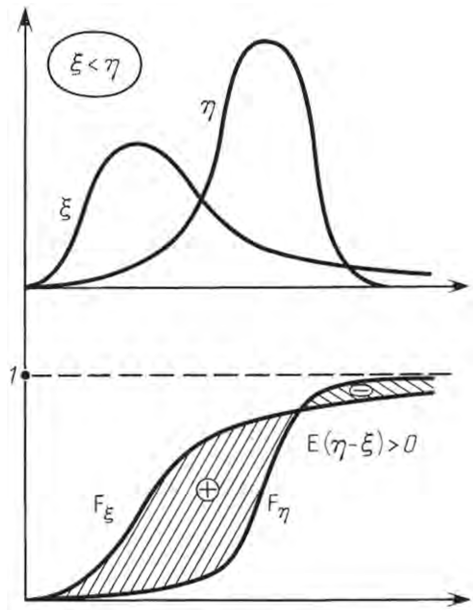

Fig. 7.1. Empirical functions of error distribution in stellar latitudes.

as drawn on a single draft. Both areas contained between the graphs have to be taken with either a plus or a minus depending on which function we find to the right and to the left of the area in question (see fig. 7.1). The distribution function $F_{t0,\,\gamma0,\,\varphi0}(x)$ that is located to the left of all the other functions $F_{t,\,\gamma,\,\varphi}$ on the average corresponds to minimal latitudinal error rates for set E. It would be natural to consider the dating t_0 and the systematic error value (γ_0, φ_0) as approximations to the real observation date and the real systematic error as made by the observer.

Let us illustrate the above with the example of another famous star catalogue dating to the second half of the XVI century and compiled by Tycho Brahe. The informative kernel that we shall be using is comprised of 13 named stars from Tycho Brahe's catalogue. We have calculated the empirical distribution functions $F_{t,\,\gamma,\,\varphi}$ for $\gamma = \varphi = 0$ and three different values of t: $t = 3$ (1600 A.D.), $t = 3.5$ (1550 A.D.) and $t = 4$ (1500 A.D.). The result can be seen in fig. 7.2. This illustration demonstrates quite well that without considering the possibility of a systematic error inherent in Tycho Brahe's catalogue ($\gamma = \varphi = 0$) epoch $t = 3.5$ proves to be the optimal dating of the catalogue (approximately 1550 A.D.). Indeed, this is the very dating for which the errors in the 13 named catalogue stars shall be minimal in the abovementioned sense. The date 1550 A.D. is really close to the known epoch when Tycho Brahe's catalogue was compiled, namely, the second half of the XVI century.

Let us provide a list of these 13 stars from Tycho Brahe's catalogue. First and foremost, they are Regulus, Spica, Arcturus, Procyon, Sirius, Lyra = Vega, Capella, Aquila and Antares, which are also included in the list of the named stars from the Almagest. Apart from that, there are four more stars: Caph = β Cas, Denebola = β Leo, Pollux = β Gem and Scheat = β Peg.

We shall now consider the empirical distribution functions $F_{t,\,\gamma,\,\varphi}$ for star set E that consists of 12 named Almagest stars (see section 1). In fig. 7.3 one sees the graphs of these functions for $t = 5$, or 1400 A.D., $t = 10$, or 900 A.D., $t = 18$, or 100 A.D., and $t = 20$, or 100 B.C. with varying values of γ. The value of φ is considered to equal zero everywhere, since the general picture is hardly affected by φ variations. The values $t = 10$, or 900 A.D., and $\gamma = 21'$ are optimal – that is to say, they generate the least serious errors.

The resulting graphical representations of the functions $F_{t,\,\gamma,\,\varphi}$ for the Almagest isn't very sensitive to changes in the contingent of the named stars. Let us cite the empirical distribution functions for all 13 stars which were used in the Tycho Brahe example, having taken the coordinates from the Almagest this time, qv in fig. 7.4. The values of $t = 10$ and $\gamma = 21'$ remain optimal for this star list as well. In fig. 7.4 one can clearly see the difference between the values of $\gamma = 21'$ and $\gamma = 0$ already pointed out above – namely, that all the graphs corresponding to $\gamma = 21'$ taken as a whole are located to the left of the graphs built for $\gamma = 0$ in general, indicating the lower error rate of the former as opposed to the latter. In other words, the value of $\gamma = 21'$ is "better" than $\gamma = 0$ for all the t dates from the a priori chosen interval.

Consideration 3. Let us conclude with discussing the issue of just how possible it is to expand the list of the named Almagest stars used as a basis for proper movement dating. Yet the coordinate precision of this expanded list (latitudinal at least) may by no means deteriorate. The first impression one gets is that the most natural way to extend the list would be including all the stars which have names of their own nowadays into it (see table P1.2 in Annex 1). Most stars received names in the Middle Ages, but this practice continued into the XVII-XIX century. It is possible that many of them were particularly significant for the Almagest catalogue compiler. We shall proceed to select just those stars from table P1.2 (from Annex 1) whose names are capitalized in the exact same manner as [1197] does it; such are the most famous of the named stars. Their number is 37, qv listed in table 7.3.

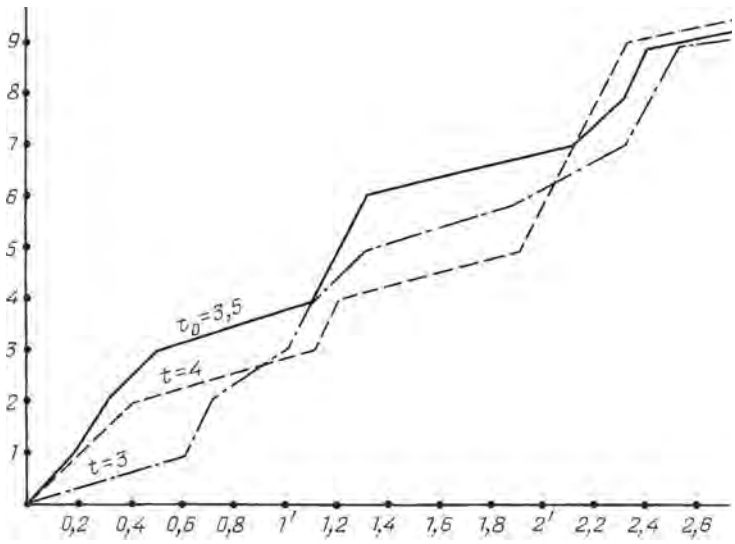

Fig. 7.2. Empirical distribution functions for Tycho Brahe's catalogue; the optimal value of $t_0 = 3.5$.

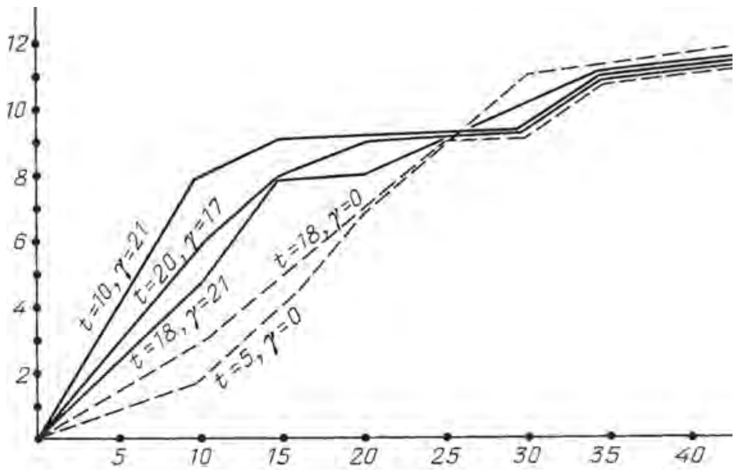

Fig. 7.3. Empirical distribution functions $F_{t,\gamma,\varphi}$ for the 12 named Almagest stars. The value of φ equals zero in every case.

However, it turns out that such an expansion of the Almagest's informative kernel drastically reduces the sample's coordinate precision, and we are particularly concerned about the latitudes being affected. Let us consider the "expanded kernel" that contains 37 Almagest stars as listed in table 7.3. Fig 7.5 demonstrates how the mean-square discrepancy behaves for these 37 stars depending on the alleged dating of the Almagest. Having calculated this discrepancy, we would allow for the variation of the systematic error's calculated rate to fluctuate within ±5' with the step value of 1 minute for parameter γ and within 30' with the step value of 1 minute for parameter β. The resultant graph demonstrates that although the minimum is reached around 400 A.D., it is very inexplicit. The minimal mean-square value roughly equals 18 minutes. If we are to allow for a variation of this value within a two-minute range, or a mere 10%, we shall end up with a "dating" interval of 1800 years, no less – between 600 B.C. and 1200 A.D. It is perfectly obvious that this result is of no interest to us, the reason being that the average precision of Ptolemy's calculations is too low for the 37-star list under consideration. It is clearly insufficient for the dating of the catalogue by proper star movements. Furthermore, this vague picture is what we get in our analysis of the latitudes, which are more precise in the Almagest catalogue, as we know. The longitudinal picture is even vaguer.

In figs. 7.6 and 7.7 one sees the dependency graphs for the quantity of stars in the extended kernel whose calculated latitudinal error does not exceed 10 and 20 minutes respectively and the presumed dating of the

No by BS4 and BS5	*Bailey's number*	*Stellar magnitude according to BS5*	$v_{\delta}(1900)$ *[1197]*	$v_{\delta}(1900)$ *[1197]*	*Stellar magnitude according to the Almagest*	*Modern name of the star and its ancient proper name as specified in caps in the Bright Stars Catalogue ([1197]), which indicates that the star in question was known very well in the past*
5340	110	–0.04	–1.098	–1.999	1	16Alp Boo (ARCTURUS)
1708	222	0.08	+0.080	–0.423	1	13Alp Aur (CAPELLA)
3982	469	1.35	–0.249	+0.003	1	32Alp Leo (REGUL)
2943	848	0.38	–0.706	–1.029	1	10Alp CMi (PROCYON)
5056	510	0.98	–0.043	–0.033	1	67Alp Vir (SPICA)
6134	553	0.96	–0.007	–0.023	2	21Alp Sco (ANTARES)
7001	149	0.03	+0.200	+0.285	1	3Alp Lyr (LYRA=VEGA)
3449	452	4.66	–0.103	–0.043	4–3	43Gam Cnc (ASELLI)
15	315	2.06	+0.137	–0.158	2–3	21Alp And (ALPHERATZ)
21	189	2.27	+0.526	–0.177	3	11Bet Cas (CAPH)
188	733	2.04	+0.232	+0.036	3	16Bet Cet (DENEDKAITOS=DIPHDA)
337	346	2.06	+0.179	–0.109	3	43Bet And (MIRACH)
617	375	2.00	+0.190	–0.144	3–2	13Alp Ari (HAMAL)
1231	781	2.95	+0.057	–0.110	3	34Gam Eri (ZAURAK)
1457	393	0.85	+0.065	–0.189	1	87Alp Tau (ALDEBARAN)
1791	400	1.65	+0.025	–0.175	3	112Bet Tau (ELNATH)
2491	818	–1.46	–0.545	–1.211	1	9Alp CMa (SIRIUS)
2890	424	1.58	–0.170	–0.102	2	66Alp Gem (CASTOR)
2990	425	1.14	–0.627	–0.051	2	78Bet Gem (POLLUX)
4057	467	2.61	+0.307	–0.151	2	41Gam1 Leo (ALGIEBA)
4301	24	1.79	–0.118	–0.071	2	50Alp UMa (DUBHE)
4534	488	2.14	–0.497	–0.119	1–2	94Bet Leo (DENEBOLA)
4660	26	3.31	+0.102	+0.004	3	69Del UMa (NEGREZ)
4905	33	1.77	+0.109	–0.010	2	77Eps UMa (ALIOTH)
4914	36	5.60	–0.238	+0.057	3	12Alp1 CVn (COR CAROLI)
5054	34	2.27	+0.119	–0.025	2	79Zet UMa (MIZAR)
5191	35	1.86	–0.124	–0.014	2	85Eta UMa (ALKAID)
5267	970	0.61	–0.020	–0.023	2	Bet Cen (AGENA)
5793	111	2.23	+0.120	–0.091	2–1	5Alp CrB (ALPHEKKA)
5854	271	2.65	+0.136	+0.044	3	24Alp Ser (UNUKALHAI)
6556	234	2.08	+0.117	–0.227	3–2	55Alp Oph (RASALHAGUE)
6879	572	1.85	–0.032	–0.125	3	20Eps Sgr (KAUS AUSTRALIS)
7557	288	0.77	+0.537	+0.387	2–1	53Alp Aql (ALTAIR)
7602	287	3.71	+0.048	–0.482	3	60Bet Aql (ALSHAIM)
8162	78	2.44	+0.150	+0.052	3	5Alp Cep (ALDERAMIN)
8728	1011	1.16	+0.336	–0.161	1	24Alp PsA (FOMALHAUT)
8775	317	2.42	+0.188	+0.142	2–3	53Bet Peg (SCHEAT)

Table 7.3. A list of fast stars possessing old names of their own according to BS4 ([1197]), all transcribed in capital letters as the most famous stars in the Middle Ages. All the celestial areas are represented here. The list is preceded by the 8 stars from the informative kernel of the Almagest, some of which don't rank among the fast stars.

Almagest. The error was calculated after the compensation of the systematic error $\gamma = 20'$. We observe fluctuations within a more or less constant value range for the entire time interval under study. A 10-minute latitudinal range covers 3-13 stars in various years, whereas about 11-16 stars wind up within the 20-minute range. These graphs give us no reliable information concerning the most probable dating of the catalogue.

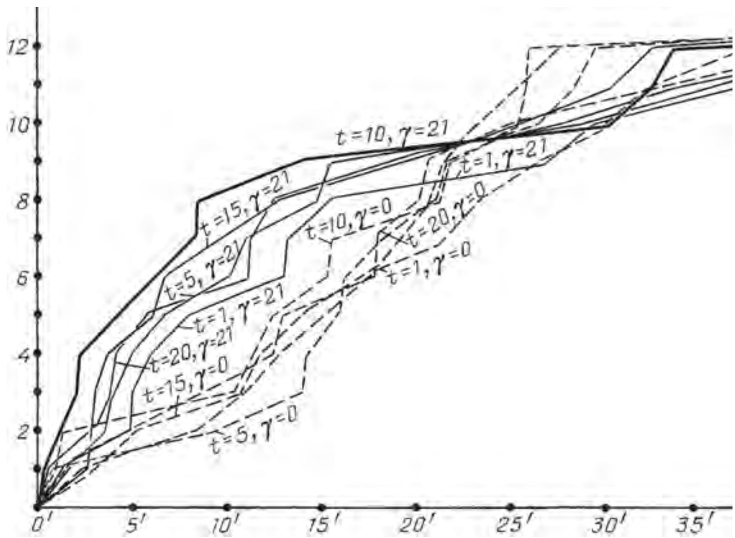

Fig. 7.4. Empirical distribution functions for the 13 bright named Almagest stars with t = 1, 5, 10, 15 and 20. Continuous lines: $\gamma = 21'$; dotted lines: $\gamma = 0$.

In fig. 7.8 we cite the mean-average discrepancy dependency graph similar to the graph in fig. 7.5. However, the only stars we took into consideration this time were the ones that got a latitudinal discrepancy of less than 30 minutes for a given dating. One sees the graph to consist of gently sloping parabola segments whose minimums fall on different years on the time axis. Thus, various parts of the 37-star list contain the valleys of respective parabolas scattered all across the historical interval.

The discovered instability of the valleys tells us that this dating method is a very imprecise one due to the fact that the valleys of many parabolas are situated at a considerable distance from the catalogue compilation date. Therefore, a variation of the stellar contingent shall distribute these valleys chaotically over the entire length of the historical interval.

In general, the graph in fig. 7.8 has its extremely poorly-manifest valley fall on the period of 700-1600 A.D., which is of zero use for a reliable dating.

We have also considered other possibilities of expanding the Almagest's informative kernel – for instance, using stellar luminosity as a criterium. Nearly all of them led to a drastic decrease in stellar coordinate precision and what can be de facto regarded as eliminating of dependency between the dating of the observations and the extended list characteristics. However, it turns out that the informative error does in fact allow a natural expansion without a drastic precision decrease. This issue is considered in detail below.

3. THE STATISTICAL DATING PROCEDURE

3.1.The description of the dating procedure

The hypothesis about the named stars of the Almagest measured in correspondence with the aberration rate of 10 minutes allows us to give a rather approximate real dating of the Almagest in section 2. We proved that the configuration of the Almagest catalogue informative kernel varies over the course of time at a high enough speed for us to determine the catalogue compilation date. Therefore one finds it makes sense to set the problem of estimating the possible dating interval.

The following procedure that we shall refer to as statistical appears to be of the most natural and obvious character; it is based on the hypothesis that the named Almagest stars were measured with a declared 10-minute latitudinal precision. Furthermore, we shall base our research on the statistic characteristics of group errors as rendered in Chapter 6. The statistical dating procedure is as follows:

A) Let us specify the confidence level $1 - \varepsilon$.

B) Now we shall consider time moment t and trust interval $I_\gamma(\varepsilon)$ for the compound $\gamma_{stat}^{ZodA}(t)$ of the group error in area *Zod A*. Now to estimate the value

$$\Delta(t) = \min \Delta(t, \gamma, \varphi), \qquad (7.3.1)$$

where the minimum is taken for all γ in $I_\gamma(\varepsilon)$ with varying φ values, while the value of

$$\Delta(t, \gamma, \varphi) = \max_{1 \le i \le 8} |\Delta B_i(t, \gamma, \varphi)|$$

defines the maximal discrepancy for all the stars from the informative kernel as calculated for the presumed dating t. Parameters (γ, φ) define a certain turn of a celestial sphere – quite arbitrarily so, as a matter of fact, qv in fig. 3.14.

C) If the educed value of $\Delta(t)$ does not exceed the declared catalogue precision rate of 10', time moment t should be regarded as the possible catalogue compilation date. Otherwise the catalogue cannot be dated to epoch t.

Quite obviously, the result of applying this dating procedure depends on the subjective choice of trust level $1 - \varepsilon$. Therefore its stability shall have to be tested against the variations of ε, which is carried out below.

3.2. The dependency of the minimax discrepancy Δ on t, γ and φ for the Almagest

We shall draw a graph for 8 of the named Almagest stars comprising the informative kernel to represent the dependency of the minimax latitudinal discrepancy $\Delta(t, \gamma, \varphi)$ on all three variables. This dependency is shown as a sequence of diagrams in figs. 7.9 and 7.10. Every diagram here corresponds to some fixed moment t. The diagrams are given for $t = 1, \dots, 18$. For other t values the respective diagrams prove void, as is the case with $t = 1$. Let us remind the reader that $t = 1$ corresponds to 1800 A.D., and $t = 18$ – to the beginning of the new era. The horizontal axes of the diagrams bear the values of γ, and the vertical – the values of φ.

Double shading marks the areas for which $\Delta(t, \gamma, \varphi) \le 10'$.

Shaded areas correspond to $10' < \Delta(t, \gamma, \varphi) \le 15'$.

The area filled with dots corresponds to $15' < \Delta(t, \gamma, \varphi) \le 20'$.

For the rest of the drawings, the expression $\Delta(t, \gamma, \varphi) > 20'$ is true. On every drawing the parameters $\gamma_{stat}^{ZodA}(t)$, $\varphi_{stat}^{ZodA}(t)$ are marked by a large dot.

The diagrams demonstrate that the "spot" with double shading that corresponds to the maximal latitudinal discrepancy of 10' for the eight named Almagest stars only exists for time moments falling into the range of $6 \le t \le 13$, or the interval between 600 and 1300 A.D.

The area with normal shading that corresponds to the maximal latitudinal discrepancy of 15' only exists for $4 \le t \le 16$. Maximal sizes of these areas are reached at $7 \le t \le 12$. For $t > 18$ the acceptable interval alteration area defined by correspondent confidence intervals contains no points where $\Delta(t, \gamma, \varphi) < 20'$. In particular, this is true for the Scaligerian dating of the epochs when Ptolemy and Hipparchus lived. Furthermore, when we attempt to date the Almagest catalogue to 100 A.D. or an earlier epoch, the latitudinal discrepancy minimax $\Delta(t)$ turns out to be two times greater than the declared 10-minute precision of the Almagest catalogue. For datings preceding 100 A.D. the value of $\Delta(t)$ exceeds even the mean-average residual error for the stars from areas *A, Zod A, B* and *Zod B*, being close to the square average residual Almagest error for celestial area *M*, or rather dim stars of the Milky Way (where the observations of such stars were complicated by the abundant stellar background which would impair their precision making its rate unacceptably low for the bright named stars). One therefore has to reject the dating of the Almagest to the epoch of roughly 100 A.D. or earlier as contradicting the Almagest catalogue.

Thus, figs. 7.9 and 7.10 demonstrate that the area permitted by the values of γ and φ fundamentally gives us no opportunity of making the latitudinal discrepancy of all 8 stars comprising the Almagest's informative kernel less than 10' for epochs preceding 600 A.D. If we are to raise the error rate threshold to 15', the earliest possible dating of the Almagest is 300 A.D.

3.3. Results of dating the Almagest catalogue statistically

Let us assign variation area $S_t(\alpha)$ of parameter γ in the following manner:

$$S_t(\alpha) = \{\gamma : \min_{\varphi} \Delta(t, \gamma, \varphi) \le \alpha\}$$

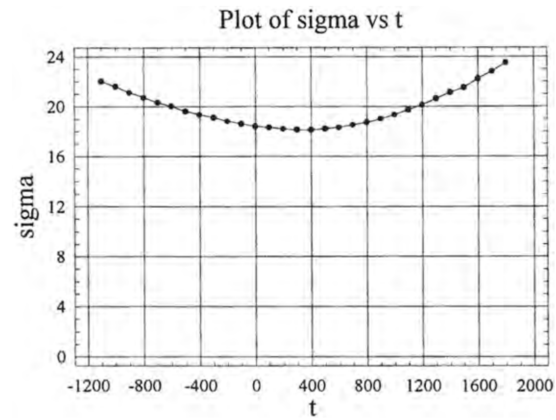

Fig. 7.5. The square average discrepancy for the 37 Almagest stars listed in Table 7.3 as the presumed dating function. The systematic error γ of the Almagest catalogue was compensated in the calculation of the discrepancy. Apart from that, the desired square average discrepancy was minimised in accordance with the variations of $\gamma = \gamma_{stat} \pm 5'$; $\beta = 0 \pm 30'$.

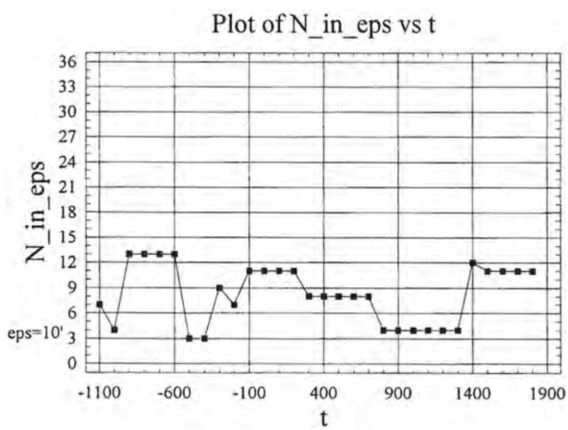

Fig. 7.6. Vertical axis: the number of Almagest stars from the list of 37 (qv in Table 7.3) whose latitudinal discrepancy doesn't exceed 10 minutes. Horizontal axis: presumed dating of the Almagest catalogue.

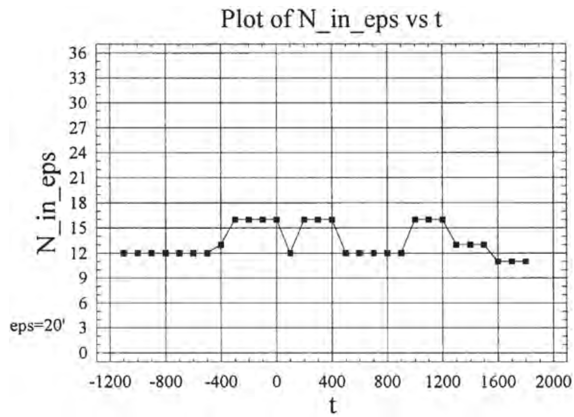

Fig. 7.7. Vertical axis: number of Almagest stars from the list of 37 (qv in Table 7.3) whose latitudinal discrepancy doesn't exceed 20'. Horizontal axis: presumed dating of the catalogue.

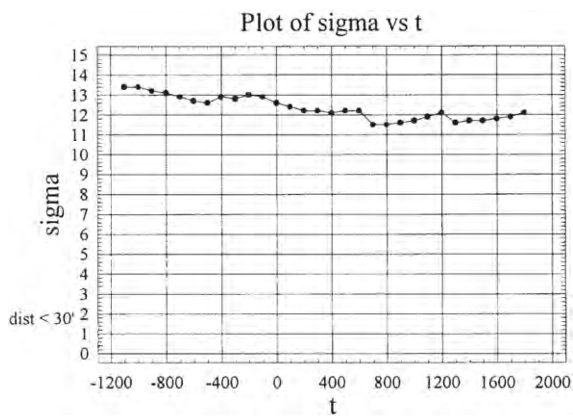

Fig. 7.8. The square average deviation for the 37 Almagest stars listed in Table 7.3, whose latitudinal discrepancy doesn't exceed 30 minutes for the presumed dating in question. The graph is built as a function of the presumed Almagest dating. In the search of the discrepancy, the catalogue's systematic error γ was compensated. Apart from that, the square average discrepancy was minimised by the variations of $\gamma = \gamma_{stat} \pm 5'$; $\beta = 0 \pm 30'$.

Set $S_t(\alpha)$ may yet turn out empty. Let us consider the intersection of set $S_t(\alpha)$ and the confidence interval $I_\gamma(\varepsilon)$ built around the value of $\gamma_{stat}^{ZodA}(t)$. If this intersection isn't empty, we can declare moment t to be the possible epoch of the Almagest catalogue's compilation in accordance with the statistical dating procedure. All of such moments t taken as a whole can be referred to as the possible dating interval of the Almagest catalogue.

The result of calculating $S_t(\alpha)$ for the Almagest is represented graphically in fig. 7.11. The dots fill the union of sets $S_t(\alpha)$ for $\alpha = 10'$. The surrounding outline corresponds to the value $\alpha = 15'$. We shall find a use for it later.

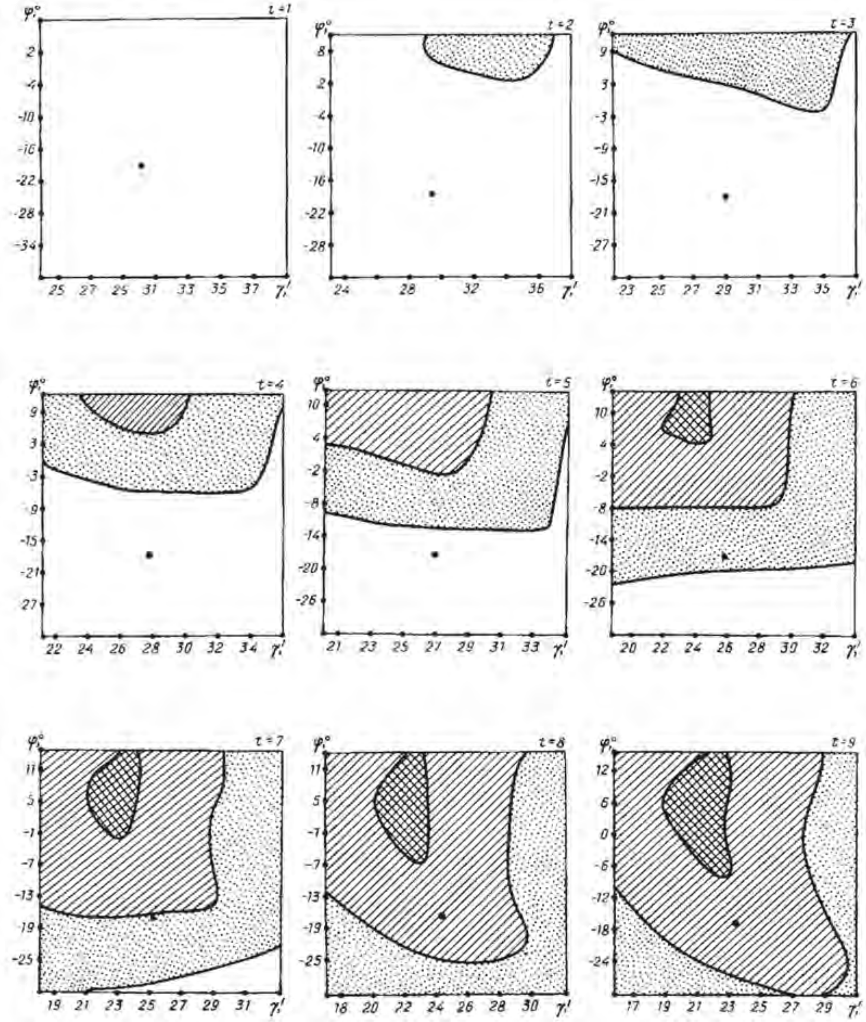

Fig. 7.9. One sees the dependency $\Delta(t, \gamma, \varphi)$ for the time values t beginning with 1, or 1800 A.D., and ending with $t = 18$, or 100 B.C. The area with double shading corresponds to $\Delta \leq 10'$. The area with single shading corresponds to $10' < \Delta \leq 15'$. The area filled with dots corresponds to $15' < \Delta \leq 20'$. The large dot corresponds to parameter pairs of $\gamma_{stat}^{ZodA}(t)$, $\varphi_{stat}^{ZodA}(t)$.

The graph of the function $\gamma_{stat}^{ZodA}(t)$ used herein was calculated in Chapter 6 (see fig. 6.8). The values of trust intervals $I_\gamma(\varepsilon)$ that correspond to different values of ε can be found in table 6.3. Fig. 7.11 implies that the possible dating interval is the same for $\varepsilon = 0.1$, $\varepsilon = 0.05$, $\varepsilon = 0.01$ and $\varepsilon = 0.005$ – namely, $6 \leq t \leq 13$.

If we are to translate the resultant dating result into regular years, we shall see that the possible dating interval in the Almagest catalogue begins in 600 A.D. and ends in 1300 A.D.

3.4. The discussion of the result

The length of the possible catalogue dating interval we ended up with equals 700 years: 1300 – 600 = 700.

The interval is a rather large one for a number of reasons. We already named the first one – the low precision of the Almagest catalogue, even if we are to accept Ptolemy's declared precision of 10'.

Such low precision makes it impossible to date the

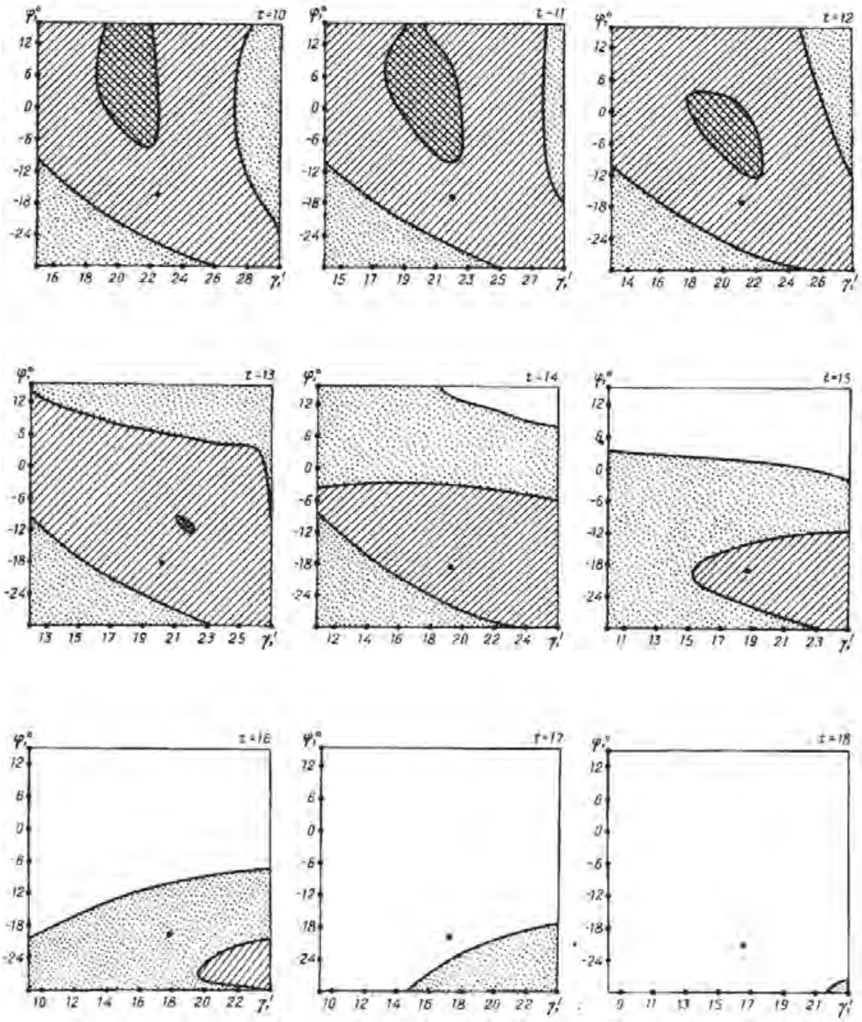

Fig. 7.10. The previous figure continued.

catalogue to a narrower time interval since even the fastest of the named stars under study (Arcturus) alters its latitude by a mere 10' every 260 years.

The value is great, and it is greater still for other kernel stars.

The second reason stems from the fact that we have only used the trust intervals of the group error's γ compound, having minimized the value $\Delta(t, \gamma, \varphi)$ by various possible values of φ, qv in formulae 7.3.1 and 7.3.2.

This approach obviously leads to the broadening of the Almagest catalogue dating interval. Indeed, if we could consider φ to be a group error like γ, we would select parameter φ from the confidence strip. This would raise the value of $\min_{\varphi} \Delta(t, \gamma, \varphi)$ and thus narrow the possible dating interval.

However, as it has been pointed out above, we do not have enough reasons to consider φ a group error in stellar groups from the Almagest that we have studied.

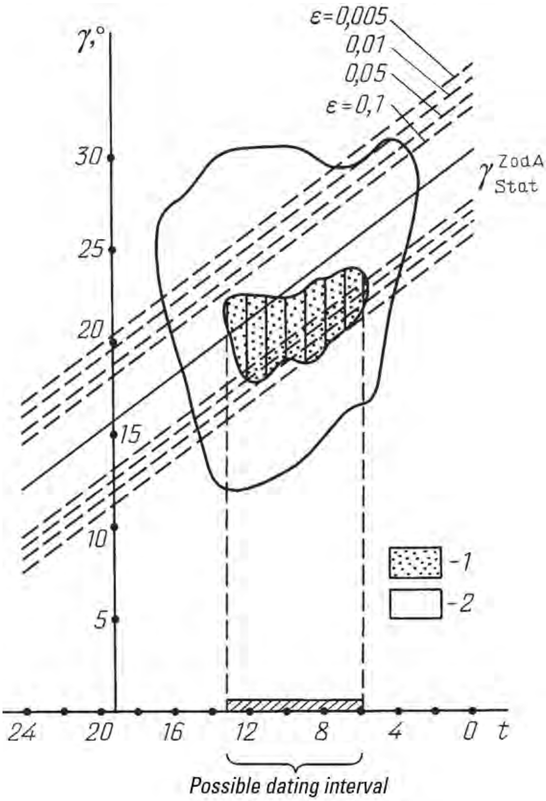

Fig. 7.11. Result of the statistical dating procedure as applied to the Almagest catalogue and using its eight named stars.

4. DATING THE ALMAGEST CATALOGUE BY THE EXPANDED INFORMATIVE KERNEL

The issue of expanding the informative kernel of the Almagest has been discussed above at the end of section 7.2. It was discovered that if we expand the kernel choosing bright and fast stars for this purpose without following any system, we cannot get an informative dating. We already understand that this is explained by the low average precision of Ptolemy's measurements, and this concerns even the bright stars. The question of what principle one could use in order to expand the 8-star informative kernel of the Almagest without the loss of latitudinal precision remains open.

We managed to solve this problem. Let us ponder the exact method used by Ptolemy in order to measure stellar latitude. It is known quite well in history of astronomy that such measurements were conducted with bright basis stars used as a "framework" of sorts which the desired stellar positions would be educed from in all the measurements to follow. The coordinates of these stars would be measured with the utmost precision and used later on. Ptolemy does not specify the exact stars that he used for basis; as we can see from the text of the Almagest, such basis stars have at least been Regulus, Spica, Antares and possibly Aldebaran (see page 247 of [1120], for instance). Three of them – namely, Regulus, Spica and Antares – have names of their own in the Almagest that employ the formula "vocatur …" ("named …"), qv above. We formulated the idea that the named stars of the Almagest received names because they served as the basis for Ptolemy's observations in the first place. This idea is confirmed by the fact that, as we have proved, the named stars of the Almagest really possess the Ptolemaic reference precision of 10' (insofar as the latitudes are concerned, at least) in areas *A, Zod A, B* and *Zod B*. This isn't true for the longitudes, but we already mentioned that it is a great deal more difficult to observe the longitudes than the latitudes. Apart from that, longitudinal precision was most probably lost when the Almagest catalogue had been re-calculated in order to correspond to other epochs. Therefore the latitudes cannot serve as a criterion of Ptolemy's real precision. It is only the latitudes that one can rely upon for this purpose.

We could prove none of the above for other celestial areas, since the systematic error rates could not be established reliably. Therefore we shall refrain from going beyond celestial areas *A, Zod A, B* and *Zod B* in our search for possible informative kernel extensions.

Let us ask about what other stars except for the basis ones – the "top ranking" stars, that is, would also be measured very well by Ptolemy? Quite naturally, the ones located in the immediate vicinity of the basis stars – the primary reason being that Ptolemy's coordinates are most likely to have followed "links" of sorts, when the coordinates of the stars close to the basis ones would be measured first, and he would proceed further taking the previously-calculated coordinates into account, step by step. Nowadays we understand that this measurement method inevitably leads to random error dispersion growth, which means greater coordinate measurement errors. The further a star is from the referential kernel, the worse it shall be measured on the average.

It would thus make sense to attempt an extension of the informative kernel, adding the stars "ranking second" thereto, which are bright enough, well-identified and located in close proximity to the basis stars. One would then have to proceed with the "third rank" of stars which are further away, the "fourth rank" which is even further and so on. If we notice this process to be accompanied by a slow decrease in average latitude precision remaining virtually the same for the basis stars and the ones closest to them, we shall ipso facto confirm our presumption that the "top ranking" stars were really included into the basis referential framework. We shall also get the opportunity to extend the "dating kernel" of the catalogue as well as checking (and, possibly, correcting) our dating.

This idea was implemented in the following manner. First of all we would have to use nothing but the stars which have perfectly sound and dependable Almagest identifications as well as observable proper movement. They are listed in table 4.3. There are 68 such stars altogether. Bear in mind that the 8-star informative kernel is included in this list in its entirety.

Eight information kernel stars were taken to represent the "top level". We have calculated the latitudinal mean-square aberration for all of them after the compensation of the systematic error. Systematic error γ was calculated in Chapter 6. We allowed for a fluctuation of this error's value within the range of ±5' with a 1-minute step. Parameter β would define the excesses within the limits of ±20' with the same step value. The mean-average discrepancy for each presumed dating of the catalogue would be selected as the minimal value achieved by said variations of parameters γ and β. The result is presented as the dependency graph of the square average discrepancy of the presumed Almagest catalogue dating. The graph built for eight of the informative kernel stars, or "top level" stars, can be seen in fig. 7.12.

The graph's minimum is reached around 900-1000 A.D. at the level of 5-6 arc minutes. This means that the guaranteed latitudinal measurement precision for Ptolemy equalled 10'-15'. Indeed, all the stars of the informative kernel are measured with the precision

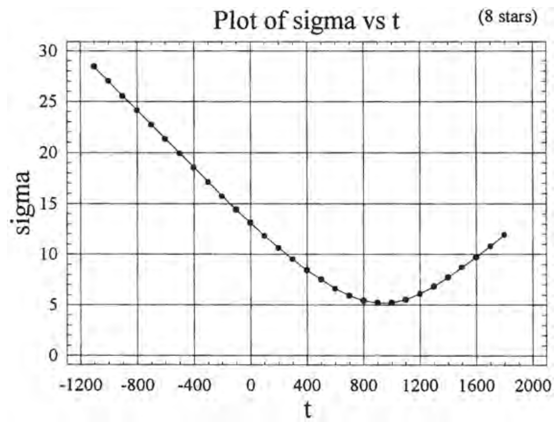

Fig. 7.12. Square average latitudinal discrepancy graph after the compensation of the systematic error for the eight "first level" stars. These eight stars comprise the informative kernel of the Almagest catalogue. According to our calculations, these very stars served as reference points in Ptolemy's observations. The square average discrepancy was minimised in accordance with the variations of parameter γ for the interval of γ_{stat} ± 5', and the variations of parameter β for the interval of 0 ± 20'. The graph reaches its minimum in 900-1000 A.D., at the level of 5-6 arc minutes. The discrepancy equals 12' for the Ptolemaic epoch of the II century A.D., which exceeds the minimum by a factor of two. The discrepancy for the epoch of Hipparchus (the II century B.C.) approximately equals 14'.

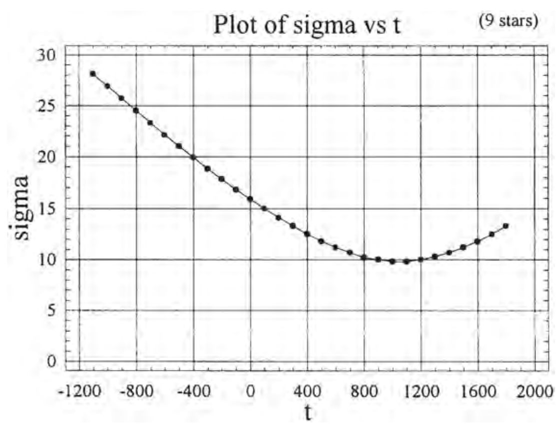

Fig. 7.13. Square average latitudinal discrepancy graph after the compensation of the systematic error for the nine "second level" stars located at the maximal distance of 5 degrees for the base ones. The square average discrepancy was minimised in accordance with the variations of parameter γ for the interval of γ_{stat} ± 5', and the variations of parameter β for the interval of 0 ± 20'. The graph reaches its minimum in 1000-1100 A.D., at the level of 9-10 arc minutes. The square average discrepancy equals 15' at least for the epoch of II century A.D. and the ones preceding it.

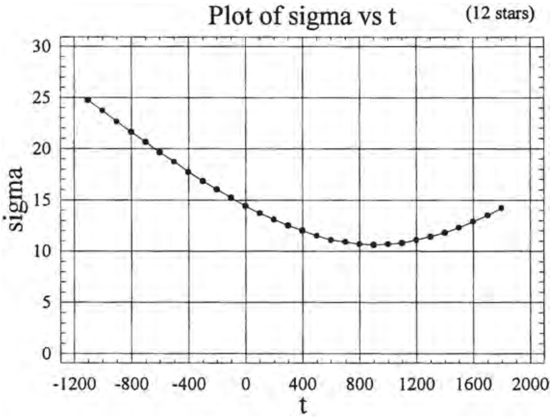

Fig. 7.14. Square average latitudinal discrepancy graph after the compensation of the systematic error for the twelve "third level" stars located at the maximal distance of 10 degrees for the base ones. The square average discrepancy was minimised in accordance with the variations of parameter γ for the interval of $\gamma_{stat} \pm 5'$, and the variations of parameter β for the interval of $0 \pm 20'$. The graph reaches its minimum in 900 A.D., at the level of 11'. The discrepancy equals 14' and more for the epoch of 100 A.D. and the ones preceding it.

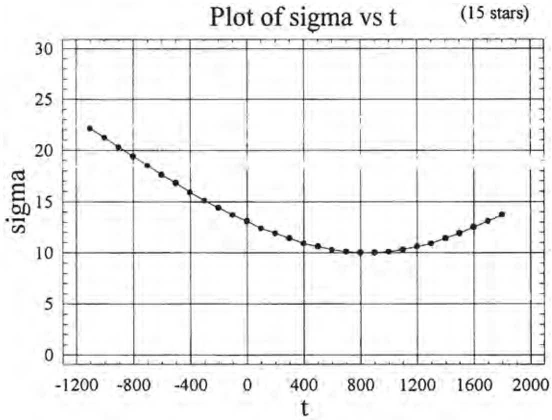

Fig. 7.15. Square average latitudinal discrepancy graph after the compensation of the systematic error for the fifteen "fourth level" stars located at the maximal distance of 15 degrees for the base ones. The square average discrepancy was minimised in accordance with the variations of parameter γ for the interval of $\gamma_{stat} \pm 5'$, and the variations of parameter β for the interval of $0 \pm 20'$. The graph reaches its minimum in 800-900 A.D., at the level of 10-11'. The discrepancy equals 12' for the epoch of 100 A.D.

of 10' or better, as we have already observed. This is in perfect concurrence with the scale grade value chosen by Ptolemy – 10'.

As for the epoch of the II century A.D., the discrepancy here reaches 12'. This is two times the permissible minimal value, which makes the early A.D. epoch completely unacceptable for the Almagest catalogue, let alone the "epoch of Hipparchus" that is supposed to have preceded it, for the discrepancy equals circa 14' for the II century B.C.

All the stars from table 4.3 were taken as the "second level" stars which are at no further distance from the closest informative kernel star than 5 degrees. There proved to be 9 such stars including the informative kernel. It turned out that we needed to add star 47δ Cnc (#3461 in catalogues BS4 and BS5). The resultant square average discrepancy graph can be seen in fig. 7.13. It is plainly visible that the picture drastically changes once we add a single star to the eight that comprise the informative kernel – and it is just one, which is close to them, well-visible to the naked eye, and isolated to boot. The reason is most likely to be that the named stars were used by Ptolemy for reference and thus were measured several times with the utmost precision. The rest of them must have been measured "following a link" from a referential star.

Nevertheless, the graph we encounter in fig. 7.13 is still informative enough. The discrepancy graph's minimum is reached around 1000-1100 A.D. at the level of 9-10 arc minutes. The square average discrepancy is substantially greater for the epoch of the II century A.D. as well as the ones preceding it. It equals 15' for 100 A.D., which is substantially greater than 150% of the minimal value.

The "third level" stars are all the stars from table 4.3 that are located at the maximal distance of 10 degrees from the informative kernel. We discovered there to be 12 such stars including the informative kernel. Apart from 47δ Cnc, the informative kernel was expanded to include 14o Leo (#3852), 8η Boo (#5235) and 26ε Sco (#6241).

The discrepancy graph is demonstrated in fig. 7.14. It hardly differs from what we had in the previous step at all. This is well understood. We are still very close to the informative kernel, which still comprises 3/4 of the total amount of stars in the sample. The graph's minimum is reached in 900 A.D. or at the level of 11'. The discrepancy for the epoch of 100 A.D. and earlier the discrepancy equals 14' or more. Judging by fig. 7.14, the most possible dating of the Almagest catalogue is the interval between the alleged years 400 and 1400 A.D.

We have taken all the "fourth level" stars from table 4.3 – the ones located at the maximum distance of 15 degrees from the informative kernel. There are 15 such stars, new additions being 78β Gem (2990), 79ζ Vir (#5107) and 24μ Leo (#3905). The discrepancy graph can be seen in fig. 7.15. The graph's minimum is reached around 800-900 A.D. at the level of 10'-11'. The discrepancy equals 12' for the epoch of 100 A.D. Thus, the value of the minimal square average discrepancy hardly alters at all. Apparently, for distances under 15° Ptolemy's tools would still allow to measure stellar coordinates against the actual basis stars, and not "following links".

Finally, for "fifth level" stars we took the ones included in Table 4.3, located at the maximal distance of 20 degrees from the informative kernel. There are 22 such stars including the informative kernel – the newcomers are 112β Tau (#1791), 60ι Gem (#2821), 68δ Leo (#4357), 29γ Boo (#5435), 3β CrB (#5747) and 5α CrB (#5793).

The discrepancy graph is shown in fig. 7.16. The graph's minimum is reached around 400-800 A.D. at the level of 22'-23'. This is the mean-square error level which is characteristic for the Almagest catalogue in general, which is to say that the effect of the basis star proximity ceases to manifest at distances of 15°-20°. The graph became almost even due to a visible decrease in measurement precision at such a distance from the basis stars. The discrepancy equals 23' for the beginning of the new era, 24' for the epoch of the V century B.C., and so on.

The last step demonstrates a drastic drop in measurement precision. The square average error rate grew by a factor of two. Therefore, before we move on in our extension of the catalogue's informative kernel, let us agree to count the square average discrepancy using only those stars for reference who get a maximal latitudinal error of 30 minutes for the assumed dating of the Almagest catalogue. This shall allow us to exclude the star which Ptolemy measured the worst from the very beginning. The choice of such stars naturally depends on the alleged dating of the catalogue. Certain alleged datings might make one star look measured well and another poorly, and vice versa.

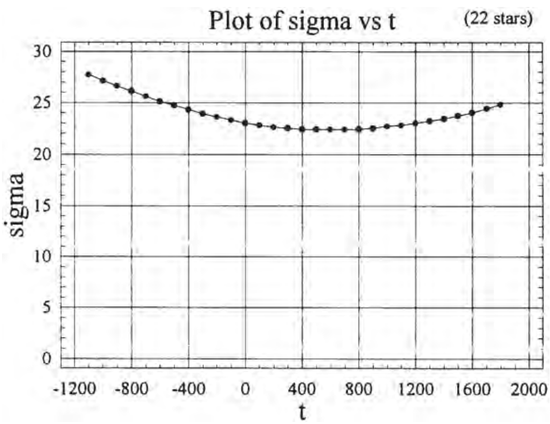

Fig. 7.16. Square average latitudinal discrepancy graph after the compensation of the systematic error for the twenty-two "fifth level" stars located at the maximal distance of 20 degrees for the base ones. The square average discrepancy was minimised in accordance with the variations of parameter γ for the interval of $\gamma_{stat} \pm 5'$, and the variations of parameter β for the interval of $0 \pm 20'$. The graph reaches its minimum in 400-800 A.D., at the level of 22-23'. This is the level that we find to be characteristic for the Almagest catalogue in general. In other words, the proximity of the "base stars" ceases to be effective at the distance of some 15-20 degrees. The graph became almost even due to the significantly lowered precision of calculations at such distance from the base stars. The discrepancy equals 23' for the beginning of the new era, 24' for the epoch of the V century B.C. etc.

We shall continue with compensating the systematic error discovered in the Almagest catalogue and make γ as well as β fluctuate within the same range as above.

The amount of stars that we find in the sample after such a selection shall be represented on the same drawing as the discrepancy. The resulting picture can be seen in fig. 7.17. One sees that the minimal square average discrepancy drops to 9' once again for 800-900 B.C., whereas the Scaligerian epoch of Ptolemy and Hipparchus, or 400 B.C. – 100 A.D., makes the discrepancy values maximal, reaching up to 12'. Let us point out that the resultant discrepancy values of 9' for the presumed dating period of 800-900 A.D. correlate very well with the discrepancy limit of 30' as specified beforehand. The matter is that the normally-distributed

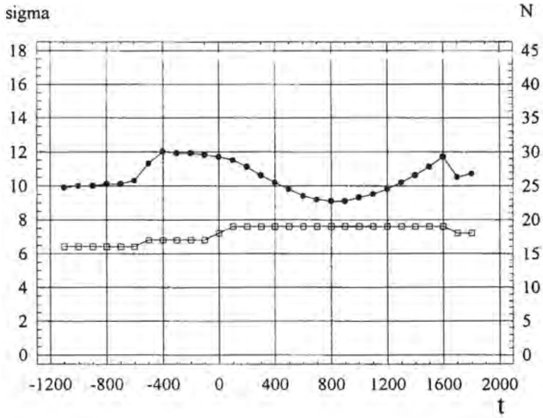

Fig. 7.17. Square average latitudinal discrepancy graph for the collected stars from table 4.3 located within 20 degrees from the stars of the catalogue's informative kernel. One can also see the graph for the number of stars in this group. The stars whose latitudinal discrepancy exceeded 30 minutes for the presumed dating in question were excluded from the sample. The systematic error of the catalogue was compensated.

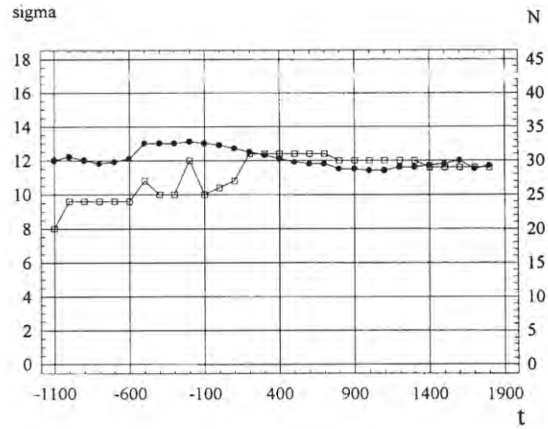

Fig. 7.19. A similar square average latitudinal discrepancy graph for the group of stars from table 4.3 located within 30 degrees from the stars of the catalogue's informative kernel. We also presented a graph for the number of stars in the group.

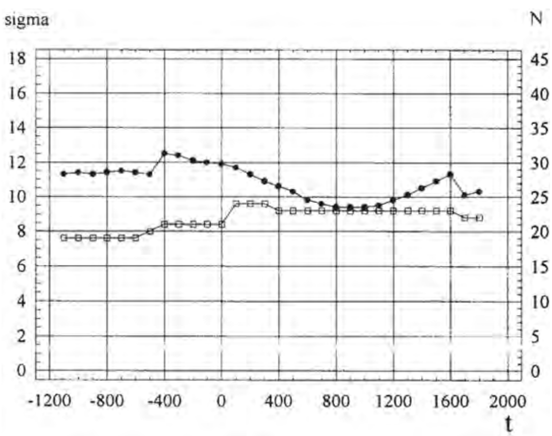

Fig. 7.18. A similar square average latitudinal discrepancy graph for the group of stars from table 4.3 located within 25 degrees from the stars of the catalogue's informative kernel. We also presented a graph for the number of stars in the group.

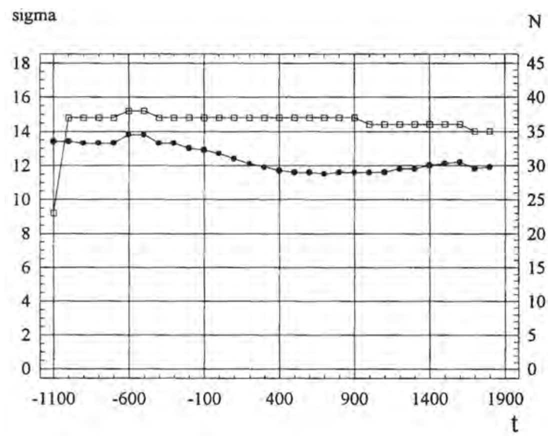

Fig. 7.20. A similar square average latitudinal discrepancy graph for the group of stars from table 4.3 located within 35 degrees from the stars of the catalogue's informative kernel. We also presented a graph for the number of stars in the group.

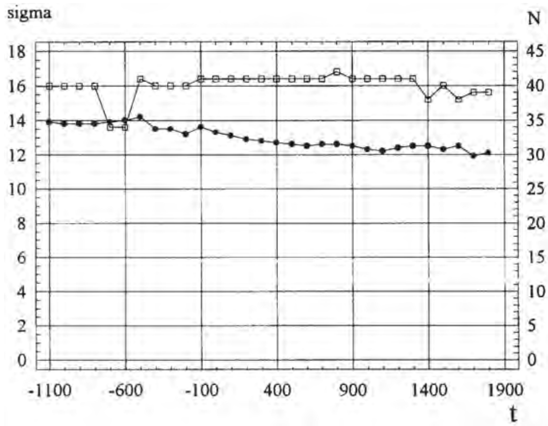

Fig. 7.21. A similar square average latitudinal discrepancy graph for the group of stars from table 4.3 located within 40 degrees from the stars of the catalogue's informative kernel. We also presented a graph for the number of stars in the group.

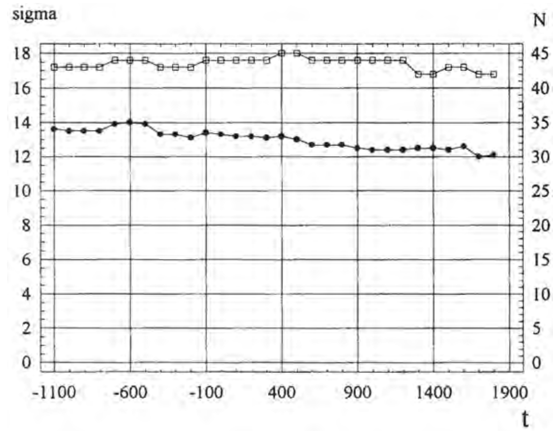

Fig. 7.22. A similar square average latitudinal discrepancy graph for the group of stars from table 4.3 located within 45 degrees from the stars of the catalogue's informative kernel. We also presented a graph for the number of stars in the group.

random value with the square average discrepancy of circa 9'-10' is likely to remain within the limits of 30' or 3σ, the probability rate being close to 1.

Let us now expand the maximal distance between the stars and the catalogue's informative kernel from 20° to 25°. We shall still only regard the stars whose latitudinal error does not exceed 30' for the presumed dating in question. See the resulting graphs in fig. 7.18 representing the discrepancy as well as the amount of stars included in the sample for each presumed dating. The square average discrepancy minimum is reached on the interval between 800 and 1000 A.D., equaling circa 9.5'. The maximal discrepancy rate is roughly equivalent to 12.5' and is reached around 400 B.C. The Scaligerian epoch of Ptolemy and Hipparchus, or the beginning of the new era, has a discrepancy rate approximating the maximum – about 12'. The amount of stars in the sample varies from 21 to 24. There are 23 stars in the sample corresponding to the minimal square average discrepancy.

We shall proceed to raise the acceptable distance between the stars and the kernel from 25° to 30°, keeping all other parameters just the same as they were. The result can be seen in fig. 7.19. Once again, the minimal possible latitudinal discrepancy can only be reached after 800 A.D. This sample contains 30 stars. The amount of stars in the sample varies between 20 and 31 stars for different presumed datings. Around the beginning of the new era the discrepancy rate is roughly equivalent to 13', which is close to the maximal value for the graph in question.

In figs. 7.20, 7.21 and 7.22 one finds similar graphs for the stars whose distance from the Almagest catalogue kernel does not exceed 35°, 40° and 45°, respectively. The sample consists of roughly 40 stars. The latitudinal square average discrepancy minimum becomes less manifest and "drifts towards the future". The graph in general begins to look more and more horizontal.

COROLLARY. Thus, the Almagest catalogue can be dated by the proper movement of a configuration of roughly 20 stars. The most possible dating interval falls on the same epoch as above, namely, 600-1200 A.D. We also discover that one has to use reliably identifiable stars which aren't located at too great a distance from the informative kernel (20°-25° maximum). If we are to exclude the stars who get a maximal 30-minute latitudinal discrepancy for alleged dating *t* from the sample, we shall end up with about 20 stars. This provides for a graph with a well-manifest minimum as

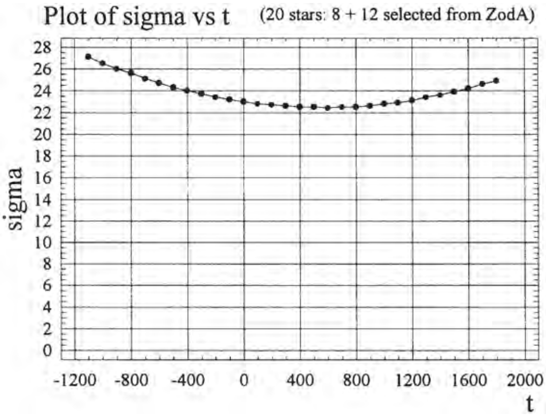

Fig. 7.23. Square average latitudinal discrepancy graph for 20 stars: 12 stars from table 4.3 located in celestial area *Zod A*, excluding the informative kernel stars, and 8 stars of the informative kernel. As one sees from the graph, the latitudinal precision for this list is substantially lower than that for the area *Zod A* on the average.

seen in fig. 7.18. The latitudinal discrepancy minimum of 9' is reached on the interval of 800-1000 A.D. The interval of 600-1200 A.D. corresponds to a discrepancy rate very close to the minimal, one of 9'-9.5'. The epoch of 400 B.C. – 100 A.D. corresponds to the maximal discrepancy rate of 11.5'-12'.

Let us emphasize that the minimal discrepancy of circa 10' can only be reached for a group of several dozen stars on the condition of their proximity to the informative kernel of the Almagest. All the other methods of selecting the stars from the combined areas *A*, *Zod A*, *B*, *Zod B* and *M* – by luminosity, "fame" etc leave us with the discrepancy minimum of roughly 20', which is typical for the Almagest in general. Remaining within a single well-measured area (*Zod A*) is also a non-option. For example, let us regard all the visibly mobile stars from this area as a whole, that is, all the stars from table 4.3 that pertain to celestial area *Zod A*. There are 12 such stars if we don't consider the informative kernel; adding the 8 stars that comprise the latter to this amount shall give us a total of 20 stars. Unfortunately, the latitude precision for this list is rather low – a great deal lower than that of area *Zod A* in general. The corresponding square average latitudinal discrepancy graph for these 20 stars as a function of the Almagest catalogue's presumed dating can be seen in fig. 7.23. The poorly-manifest minimum corresponds to the level of 23'. It is reached on the interval between 400 and 800 A.D. A mere 1' above the minimum, and we shall cover the entire interval of 400 B.C. and 1500 A.D. Therefore, this list doesn't permit any reliable datings due to the low average precision of the stellar latitudes that it contains. Even the eight informative kernel stars cannot improve the average latitudinal precision of this list owing to the fact that most of the visibly mobile stars from area *Zod A* are rather dim, and were therefore measured rather badly by Ptolemy on the average. Bear in mind that the average precision of his latitudinal measurements equals 12'-13' for the entire *Zod A* area, which is a lot better than the 23' that we get for the 20 stars in question.

We have thus managed to expand the informative kernel of the Almagest without any substantial precision losses to 15 reliably and unambiguously identifiable Almagest stars that are also visibly mobile, by which we mean that their minimal annual proper movement speed equals 0.1" by one of the coordinates at least. The choice of the celestial coordinate system is of little importance here, and so we are using the 1900 A.D. equatorial coordinates for the sake of convenience, since they are used in the modern star catalogues that we have used. Let us now cite the final list of the 15 stars that enable a proper movement dating of the Almagest. The BS4 number of the star is specified in parentheses ([1197]).

1) 16α Boo (5340); 2) 13α Aur (1708); 3) 32α Leo (3982); 4) 10α CMi (2943); 5) 67α Vir (5056); 6) 21α Sco (6134); 7) 3α Lyr (7001); 8) 43γ Cnc (3449); 9) 78β Gem (2990); 10) 47δ Cnc (3461); 11) 14o Leo (3852); 12) 24μ Leo (3905); 13) 79ζ Vir (5107); 14) 8η Boo (5235); 15) 26ε Sco (6241).

5. DATING THE ALMAGEST CATALOGUE BY A VARIETY OF 8-STAR CONFIGURATIONS CONSISTING OF BRIGHT STARS

The idea behind this calculation as well as the calculation itself are credited to Professor Dennis Duke from the State University of Florida, an eminent specialist in data analysis. He suggested to study all possible configurations of eight named Almagest stars.

Professor Duke chose a set of 72 stars whose Almagest magnitude is less than 3 (bear in mind that the lower the value, the brighter the star) for this purpose. Then he selected all the 8-star combinations from this number whose maximal latitudinal error in the Almagest catalogue does not exceed 10' for a certain non-zero time interval (t_1, t_2) that covers the entire period between 400 B.C. and 1600 A.D. The total amounted to 736 eight-star combinations out of 500.000 possibilities. Each one of these combinations specifies a dating interval (t_1, t_2) of its own. Professor Duke studied the set of such "dating interval centres", or the set of values $(t_1 + t_2) / 2$. It turns out that if one is to build a frequency distribution histogram of these centres on the time axis, one sees a manifest maximum on the interval of 600-900 A.D., qv in fig. 7.24. Therefore, the epoch of the VII-X century A.D. is the most likely date when the Almagest catalogue was compiled.

The approach suggested by Professor Duke has the advantage that poorly-measured or excessively slow stellar configurations are automatically excluded from the sample due to the fact that their dating intervals are either void for the 10-minute latitudinal threshold, or great enough to go well beyond the historical interval of 400 B.C. – 1500 A.D. as chosen by Professor Duke a priori. It turns out that after such a rigid selection one is still left with a great many configurations, namely, 736 of them, each one containing eight stars. If we are to chose the "dating interval centre" of some such configuration as a dating with a latitudinal level of 10', we shall end up with the Almagest catalogue dating that shall contain some random error, or a perturbed catalogue compilation dating. Once we build a distribution graph of these perturbed datings, we shall be able to date the Almagest catalogue with a great deal more precision than in case of using a single configuration.

The natural assumption is that the true dating of the catalogue equals the average value of the randomly perturbed datings. This average can be estimated by the empirical distribution that we have at our disposal. Considering the true perturbation distribution to be close to normal, it is easy to estimate its dispersion. The selective mean-square distribution aberration as seen in fig. 7.24 roughly equals 350 years. Seeing as how the sample was censored in accordance to an a priori chosen time interval that proved asymmetric in relation to the distribution centre (qv in fig. 7.24), the average estimation for this distribution turns out to be shifted sideways. If we are to take this effect into consideration, the more accurate estimate of the mean-square aberration shall yield an even smaller value.

Moreover, the centre of the selective distribution is located near the year 800. Had the sample elements been independent, one could come to the conclusion that the real dating of the Almagest catalogue compilation can be located within

$$800 \pm (3 \times 400) / \sqrt{736},$$

or 800 ± 45 years. However, one cannot consider the sample elements to be independent since the real precision of the 800 A.D. dating for the Almagest is a great deal lower than ±45 years. Nevertheless, the early A.D. period dating or an even earlier one can be regarded as highly improbable in this situation, and all but out of the question.

6.
THE STATISTICAL PROCEDURE OF DATING THE ALMAGEST CATALOGUE: STABILITY ANALYSIS

6.1. The necessity of using variable algorithm values

The implementation of the dating procedure as described above involved a rather arbitrary choice of certain values defining the algorithm, whereas other values result from statistical conclusion. One therefore has to check the behaviour of the resultant dating interval in case of said values being subject to alteration.

6.2. Trust level variation

The value of ε that determines the trust level was chosen rather arbitrarily. Bear in mind that in statistical problems it represents the acceptable error probability rate, that is, $\varepsilon = 0.1$ stands for the error probability rate of 0.1. The smaller the value of ε, the greater the trust interval. The dependency of the trust interval size on ε is studied in chapters 5 and 6 – see table 6.3 in particular.

Let us now consider the variation of our dating in-

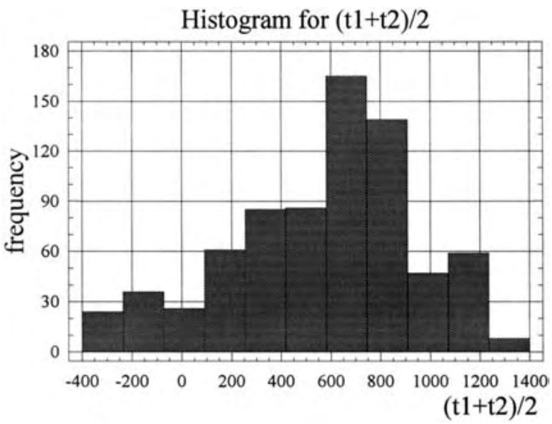

Fig. 7.24. Frequency distribution histogram for the "dating interval" centres of 736 bright Almagest star configurations of 8. One can see the peak manifest at the interval of 600-900 A.D.

terval in accordance with ε. We already mentioned that every value of ε that is less than 0.1 gives us the same dating interval for the Almagest catalogue, and this is also implied by fig. 7.11. This results from the $S_t(\alpha)$interval position where $\alpha = 10'$.

However, let us see whether we should come up with an altogether different picture if we are to choose a different guaranteed precision value α for the Almagest catalogue that will not equal 10 minutes as declared by Ptolemy. Let us consider α to equal 15' (see the corresponding shaded area in fig. 7.11). The possible dating interval of the Almagest catalogue shall naturally expand. The upper threshold of the expanded interval does not depend on ε and equals $t = 3$, or 1600 A.D. The lower threshold is only marginally dependent on ε, namely, it equals $t = 16.3$ for $\varepsilon = 0$, or 270 A.D., whereas $\varepsilon = 0.005$ shall yield $t = 16.5$ – 250 B.C., in other words.

These results therefore demonstrate that the subjective choice of trust level ε hardly affects the value of the lower threshold of the Almagest catalogue's possible dating interval.

We have also discovered how the size of the dating interval is affected by the value of α whose meaning represents the latitudinal measurement precision of the catalogue's named stars – in particular, even raising the value from the precision rate of 10' as declared by Ptolemy to 15', or making it greater by a factor of 1.5, the resultant dating interval of the Almagest catalogue does not include the Scaligerian epoch of Ptolemy, let alone Hipparchus.

6.3. Reducing the contingent of the Almagest catalogue informative kernel

The choice of the catalogue's informative kernel is also subjective to a great extent. Indeed, we have discarded 4 named stars out of 12 – Canopus, Previndemiatrix, Sirius and Aquila = Altair. If the rejection of the first two stars is explained by reasons which are of an extraneous nature insofar as our research is concerned, Sirius and Aquila were rejected due to the fact that the group errors for their respective surroundings fail to coincide with the group error for *Zod A*. However, in Chapter 6 we demonstrate that there are at least two more stars – namely, Lyra and Capella, for which the group errors of their surroundings fail to correspond with the group error for *Zod A*. The previous presumption is of a rather arbitrary nature, since we cannot determine these errors. Apart from that, these two stars are located at a considerable distance from the Zodiac, close to the relatively poorly-measured celestial region *M*.

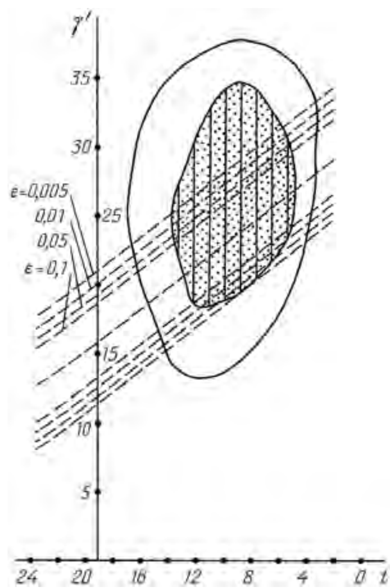

Fig. 7.25. A result of the statistical procedure that involved the dating of the Almagest catalogue by 6 of its named stars.

Let us now ponder the possible dating interval of the Almagest catalogue as it shall be if we exclude these two stars and leave just six of them in the informative kernel of the catalogue, namely, Arcturus, Regulus, Antares, Spica, Aselli and Procyon. We can see the result in fig. 7.25 (similar to fig. 7.11). Although the value area of parameter γ for which the maximal latitudinal discrepancy does not exceed the level of 10' or 15' has grown substantially, the boundaries of the possible dating interval only changed very marginally. The top boundary remains the same for both levels; the lower boundary for the 15-minute level remains the same as compared to the one we get when we consider the eight kernel stars. The lower boundary for α = 10' moved backwards in time by a mere 100 years.

Thus, if we are to take into account nothing but the 6 named stars of the Almagest catalogue from area *Zod A* or its immediate vicinity, we can come to the conclusion that the Almagest star catalogue could not have been compiled earlier than 500 A.D.

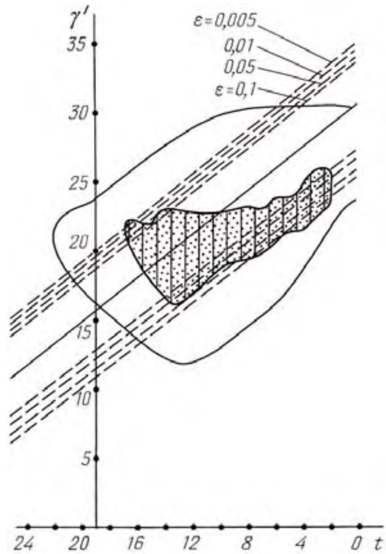

Fig. 7.26. A result of the statistical procedure that involved the dating of the Almagest catalogue by 7 of its named stars.

6.4. The exclusion of Arcturus does not affect the dating of the Almagest catalogue substantially

We are confronted with yet another question. Could the Almagest catalogue dating interval that we have calculated be the result of just one star moving? This question does make sense, since if we are to find such a star, the possible error in how its coordinates were measured can distort the resultant dating. The only candidate for such role of a "dating star" in the informative kernel is Arcturus. It is the fastest of all eight stars, and it defines our dating interval to a large extent. The stars that surround it weren't measured very well, qv in Chapter 6. Therefore, if the individual coordinate error for Arcturus is great enough, the possible dating interval can become rather distorted. Let us check what this interval shall be like if we exclude Arcturus from the informative kernel of the Almagest catalogue, limiting it to just seven stars. The length of the new interval shall naturally extend, since it is basically inversely proportional to the maximum stellar speed of the catalogue's informative kernel. We can see the result as a diagram in fig. 7.26, which demonstrates clearly that even with the fastest star of the informative kernel (Arcturus) absent, the 10-minute area does not go further back in time than 300 A.D. ($t = 16$) at the trust level of $1 - \varepsilon = 0.95$ or lower. It is only if we are to extend the confidence strip to $1 - \varepsilon = 0.99$, or 99%, that this area begins to cover 200 A.D., which is to say that the Scaligerian epoch of Ptolemy is not included into the dating interval, let alone the even more ancient Scaligerian epoch of Hipparchus.

Let us now consider the 15-minute area. It reaches 100 B.C. ($t = 20$) at the trust level of $1 - \varepsilon = 0.95$. Trust level of $1 - \varepsilon = 0.99$ allows to reach 200 B.C. – therefore, the Scaligerian epoch of Ptolemy is only covered if we are to make the conditions extremely lax.

One wonders whether the trust level of $1 - \varepsilon = 0.95$ is sufficient in our case. Apparently so, since the precision defined by a level of 95% is high enough for historical research; actually, such values are considered acceptable for technical applications as well, and those require a very high level of precision indeed. Let us cite [273] for reference, which is a work concerned with the dating of the Almagest, for which we have chosen the value of $\varepsilon = 0.2$ making the confidence interval a mere 80%. Therefore, our conclusions do have a very high degree of reliability.

We can conclude saying that neither the change of trust level, nor the alterations in the contingent of the informative kernel, nor the variation of the guaranteed measurement precision value can affect the primary conclusion that we made, namely, that the Almagest catalogue was compiled a great deal later than I-II century A.D., which is the Scaligerian epoch of Ptolemy.

7. THE GEOMETRICAL DATING OF THE ALMAGEST

The conclusions that we came to in sections 2-6 have all been of a statistical character. The actual group error values were determined with some statistical error. Therefore, the conclusions regarding the group error coincidence for various Almagest constellations can be false, albeit this probability is very low indeed, since we analysed the stability of our statistical result in the previous section. However, in order to guarantee the absence of statistical errors, let us set statistics aside for a while and turn to purely geometrical considerations.

Let us consider the "minimax latitudinal discrepancy" for the previously defined informative kernel of the Almagest catalogue that consists of 8 named stars:

$$\delta(t) = \min \Delta(t, \gamma, \varphi), \qquad (7.7.1)$$

where the minimum is selected according to various values of γ and φ, and then compare this equation to 7.3.1. The sole difference between them is the altered value range of parameter γ. In formula 7.3.1 γ would change inside the scope of the confidence strip that covers point $\gamma_{stat}(t)$. Equation 7.7.1 contains no such limitation; therefore, $\delta(t) \leq \Delta(t)$.

Let us use $\gamma_{geom}(t)$ and $\varphi_{geom}(t)$ to represent the values of γ and φ that comprise the minimum of the right part (7.7.1). Possible low precision of the $\gamma_{geom}(t)$ and $\varphi_{geom}(t)$ estimation procedure is of little importance here.

Let us recollect the situation we already encountered in Section 3 where we removed the limitations from parameter φ. These limitations only concerned γ. As we have seen, it leads to a dating interval that remains unaffected by the statistical estimation char-

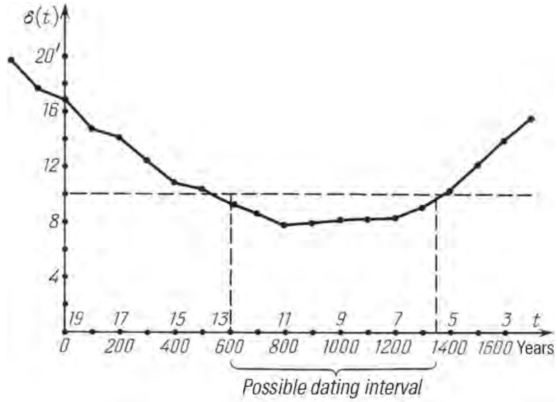

Fig. 7.27. Geometrical procedure of dating the Almagest catalogue: $\delta(t) = \Delta_b(t, \gamma_{geom}(t), \varphi(t))$.

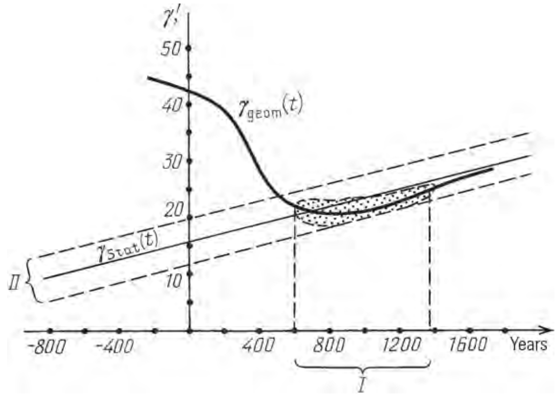

Fig. 7.28. $\gamma_{geom}(t)$ dependency graph together with the trust interval.

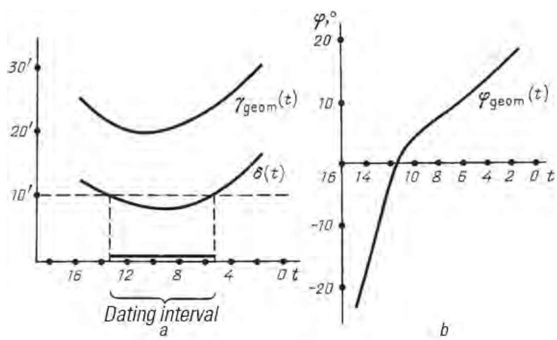

Fig. 7.29. The geometrical dating procedure of the Almagest catalogue.

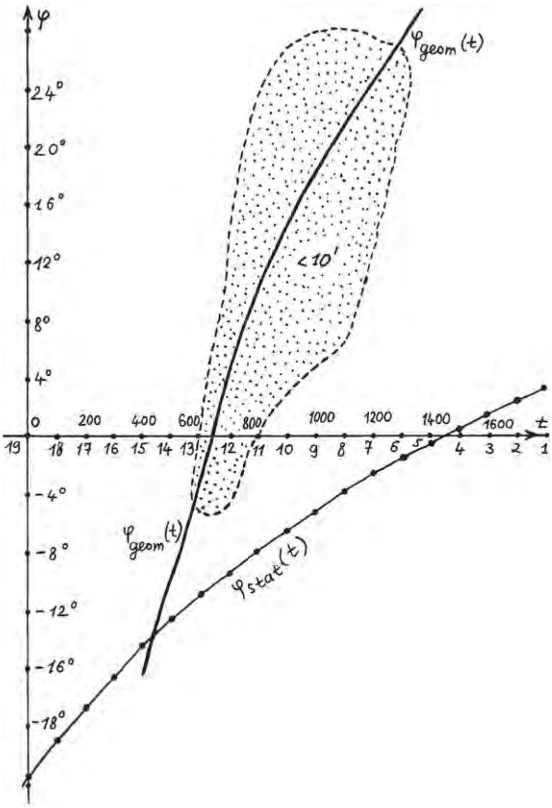

Fig. 7.30. The geometrical dating procedure of the Almagest catalogue.

acteristics of φ. The interval is nonetheless a large one. We shall do something of the kind with both parameters (γ, φ). The values of $\gamma_{geom}(t)$ and $\varphi_{geom}(t)$ that we have introduced can be considered parameters defining the group error of the catalogue's informative kernel, provided the catalogue was compiled in a certain epoch t.

Taking all of the above into account, let us consider the possible dating interval of the catalogue to be all of these time moments t taken as a whole, for which $\delta(t) \leq 10'$. In order to find this interval, let us draw the graph of $\delta(t)$ in figs. 7.27, 7.28, 7.29 and 7.30., as well as the graphs of the functions $\gamma_{geom}(t)$ and $\varphi_{geom}(t)$. The resulting graph of $\delta(t)$ was built according to the formula 7.7.1, and the values of $\Delta(t, \gamma, \varphi)$ were calculated by 7.3.1, with the subsequent sorting out by γ and φ. For comparison, we can study the $\varphi_{geom}(t)$ dependency graph in fig 7.28 complete with the confidence strip (see section 6). One also sees the area of such values of (t, γ) for which $\Delta(t, \gamma, \varphi) < 10'$ with a certain value of φ.

According to these graphs, the previously estimated Almagest catalogue dating interval does not expand even if we are to use a geometrical dating procedure. This is additional proof to the fact that our statistical estimations of γ_{stat}^{ZodA} calculated for the majority of the Almagest catalogue stars do in fact correspond to the group error in the small array of named Almagest stars. Apart from that, we prove that there is no option to combine the real celestial sphere with the Almagest stars in such a way that all the stars would have a latitudinal discrepancy of less than 10' anywhere outside the interval between 600 A.D. and 1300 A.D.

We shall conclude with citing the presumed dating t dependency graphs for the individual latitudinal discrepancies of all 8 stars from the informative kernel of the Almagest at fixed values of $\gamma = 20'$ and $\varphi = 0$ (see fig. 7.31). The upper envelope of these graphs is similar to the curve in fig. 7.25 that represents the dependency of the minimal discrepancy on the presumed dating t for the greater part of the time interval after 0 A.D. $(0 < t < 9)$. This results from the value of $\gamma = 20'$ being close to that of $\gamma_{geom}(t)$, whereas $\varphi = 0$ is close to $\varphi_{geom}(t)$ for the greater part of this interval. The result is not particularly sensitive to the variation of the φ value.

Fig. 7.31 demonstrates which exact stars of the Almagest catalogue's informative kernel allow to reach the minimal value of the latitudinal discrepancy $\delta(t)$ for different presumed datings t. In fig. 7.31 one can plainly see the concentration of zero latitudinal discrepancy values near $t = 10$, or approximately 900 A.D. This presumed catalogue dating virtually eradicates the discrepancies for three informative kernel stars simultaneously, namely, Arcturus (α Boo), Regulus (α Leo) and Procyon (α CMi). For all the other informative kernel stars of the Almagest catalogue it is only the latitudinal discrepancy of Aselli (γ Can) that reaches zero near the beginning of the new era.

It would be interesting to examine a possible link between the abovementioned zero discrepancy concentration and the fact that Arcturus and Regulus, as well as Sirius, occupied an exceptionally important

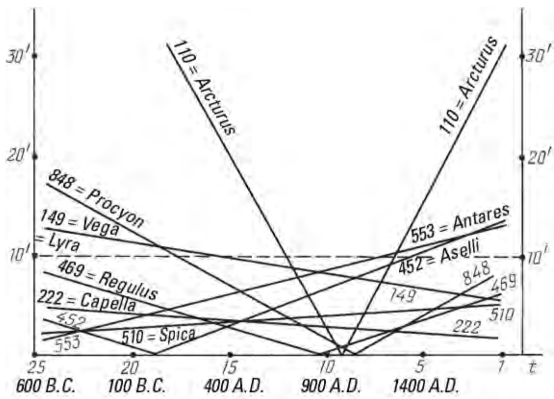

Fig. 7.31. Individual latitudinal discrepancies of the Almagest catalogue with $\beta \approx 0'$, $\gamma \approx 21'$.

position in "ancient" astronomy. Arcturus, for instance, must have been the first star to have received a name of its own in "ancient" Greek astronomy, being the brightest star of the Northern hemisphere. It is mentioned in an "ancient" poem by Aratus that contains references to the celestial sphere. Regulus is the star that was used for reference for measuring the coordinates of all other stars and planets in Greek astronomy.

8. THE STABILITY OF THE GEOMETRICAL DATING METHOD APPLIED TO THE ALMAGEST CATALOGUE. The influence of various astronomical instrument errors on the dating result

8.1. Poorly-manufactured astronomical instruments may have impaired the measurement precision

The geometrical dating method does not contain trusted probability factor ε. However, one has to test its stability in relation to the declared catalogue precision as well as the informative kernel contingent. The conclusions we come to here are similar to the ones of section 6 to a large extent. Thus, raising the precision level from 10' to 15' leads to shifting the lower boundary of the dating interval back to 250 A.D. The dating interval for the compacted informative kernel of 6 stars which are either located in area *Zod A* or in its immediate vicinity also only grew by a mere 100 years, becoming 500 A.D. – 1300 A.D. Once we remove the fast Arcturus from the informative kernel of the catalogue, the dating interval expands to 200 A.D. – 1600 A.D.

Therefore, the Almagest catalogue dating interval as estimated by a geometrical procedure fails to cover the Scaligerian epoch of Ptolemy, let alone the Scaligerian Hipparchus.

Apart from that, we shall demonstrate the stability of the geometrical dating procedure under the possible influence of astronomical instrument errors.

The geometrical dating method is based on accounting for the observer's error in the ecliptic pole estimation. All the possible rotations of the sphere, or, in other words, the orthogonal rotation of the coordinate grid in space, are taken into account. If we're interested in nothing but the latitudes, the rotation of the sphere can be defined solely by the pole shift vector, since the residual rotation component does not affect the latitudes.

Let us assume the pole shift vector to have the coordinates of (γ, φ). If we can make the sphere rotate in such a manner that will reduce the maximal latitudinal discrepancy (of the informative kernel of the catalogue, or the zodiacal stars contained therein, for instance, and so on) to a value lower than that of Δ, the dating of the catalogue is a feasibility. Let us remind the reader that for the Almagest catalogue $\Delta = 10'$.

In all of the cases considered above, orthogonal rotations of the celestial sphere sufficed in order to make the maximal latitudinal discrepancy lower than the declared precision rate of catalogue Δ, ipso facto dating the catalogue and also confirming the precision of Δ as declared by Ptolemy. However, we have so far left the fact that Ptolemy might have used an imperfect astronomical instrument out of consideration. An example could be an astrolabe with metallic rings with a slight aberration of the perfect circular shape. A ring could be oblate from one end and stretched from another. Apart from that, some of this instrument's planes could be not quite as perpendicular in reality as they should have been ideally. Some of the angles could become warped as a result and give somewhat different scales on different axes.

In other words, the instrument, as well as the coordinate grid that it would define in three-dimensional space, could be subject to a certain deformation. It could affect the measurement results setting them off the mark. One is well entitled to wonder about how minor deformations of the instrument – or, in other words, the coordinate grid that said instrument corresponds to, influence the result of the measurement. How great should the instrument's distortions be to substantially impair the results of the observations? We answer all of these questions below.

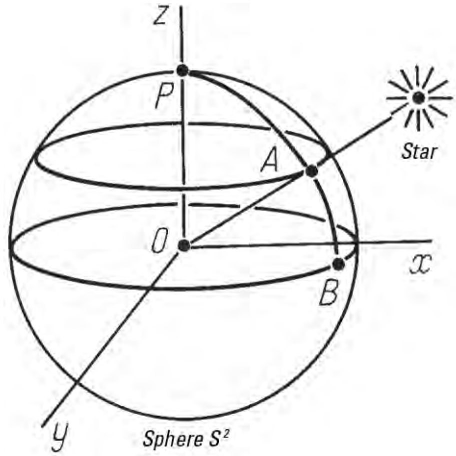

Fig. 7.32. The calculation of a star's ecliptic latitude.

8.2. Formulating the problem mathematically

Let us formulate the problem in precise mathematical terms. We shall consider a three-dimensional Euclidean space whose centre contains a sphere that corresponds to three mutually orthogonal coordinate axes. These axes define pairs of orthogonal coordinate planes. In order to measure ecliptic stellar coordinates, one would have to project the star from the beginning of the coordinate scale into point *A*, qv in fig. 7.32. The resultant point *A* on the sphere is defined by its coordinates – spherical, for instance. These coordinates are then included into the observer's catalogue.

Let us now consider the axis *z* to be directed at the ecliptic pole *P*, whereas plane *xy* crosses the ecliptic of the sphere. We have already made a detailed explanation of the fact that stellar latitudes are the most reliably measured coordinate. Therefore it is the latitude of star *A* that shall be of the utmost interest to us. The latitude is measured across the meridian that connects ecliptic pole *P* to star *A*. Zero latitude is the ecliptic itself, or parallel zero. In fig. 7.32 the ecliptic latitude of star *A* is measured by the length of arc *AB*.

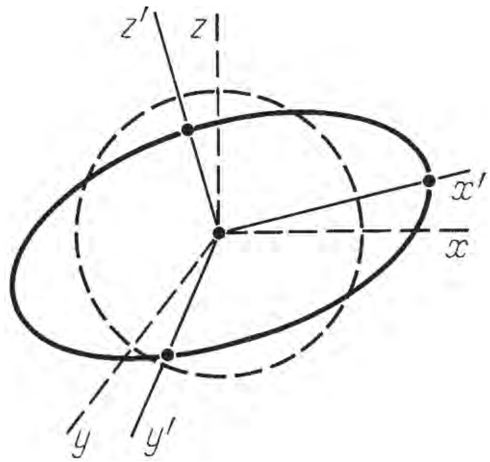

Fig. 7.33. The transformation of a sphere into an ellipsoid under the influence of minor linear deformation of the ambient space.

The process of inclusion of stellar coordinates as described above has the implication that the observer's instrument creates an ideal spherical coordinate system in the surrounding three-dimensional space. However, the real instrument might be somewhat deformed. Disregarding the second-order effects and without loss of generality in any way one can consider the instrument's deformation to cause some sort of linear space transformation of the Euclidean coordinate system. It would be natural to consider this linear transformation close to an identical case, since too great a distortion would be noticed by the observer who claims the precision of 10', as we have already seen. Even if the deformation of the coordinate system contains small non-linear perturbations, we are de facto considering the first linear approximation that describes the instrument's distortion.

A linear transformation of three-dimensional space that leaves the beginning of the coordinates intact is specified by the matrix

$$C = \begin{pmatrix} c_{11}c_{12}c_{13} \\ c_{21}c_{22}c_{23} \\ c_{31}c_{32}c_{33} \end{pmatrix}$$

This transformation distorts the original Euclidean coordinate system. Elementary quadratic form theory tells us explicitly that non-degenerate linear transformation close to the identical deforms a sphere making it an ellipsoid of sorts, qv in fig. 7.33. Thus, although the original mutually orthogonal coordinate lines are somewhat shifted, ceasing to be orthogonal, one can always find three new mutually orthogonal lines aligned along the ellipsoid axes. These three new lines are indicated as x', y' and z' in fig. 7.33.

Thus, the ends of our research allow us to assume that the linear transformation deforms the sphere in the following manner: the first thing that happens is some kind of turn (orthogonal transformation) that turns the mutually orthogonal axes x, y and z into new mutually orthogonal axes x', y' and z'. This last transformation is specified unambiguously by the diagonal matrix

$$R = \begin{pmatrix} \lambda_1 0 \ 0 \\ 0 \ \lambda_2 0 \\ 0 \ 0 \ \lambda_3 \end{pmatrix}$$

Stretching coefficients λ_1, λ_2 and λ_3 represent certain real numbers which can be positive or negative, but the very concept of the problem implies that they differ from zero.

8.3. The deformation of a sphere into an ellipsoid

The deformations of the coordinate grid which were caused by orthogonal turns have been studied above, and so one can now concentrate all of one's attention on the second transformation, namely, the transformation of the similarity defined by diagonal matrix R.

Thus, without loss of generality we can assume the deformation of the astronomical instrument that spawns a linear transformation of the three-dimensional Euclidean coordinate grid is specified by similarity transformation R with stretching coefficients λ_1, λ_2 and λ_3, qv in fig. 7.34. Let us point out that the values of λ_i can equal one, be greater than one or smaller than one independently from each other. Therefore when we are referring to stretching coefficients, in reality it isn't just the factual stretching (or linear size expansion along the axis), but also the possible compression, or linear size reduction. If λ_i is greater than 1 for some i, we have expansion; if its value is smaller than one, we observe compression to take place on the axis in question.

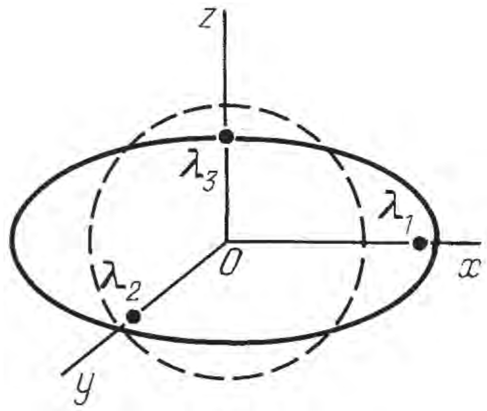

Fig. 7.34. The transformation of the similarity with independent coefficients of expansion or compression along the three orthogonal axes.

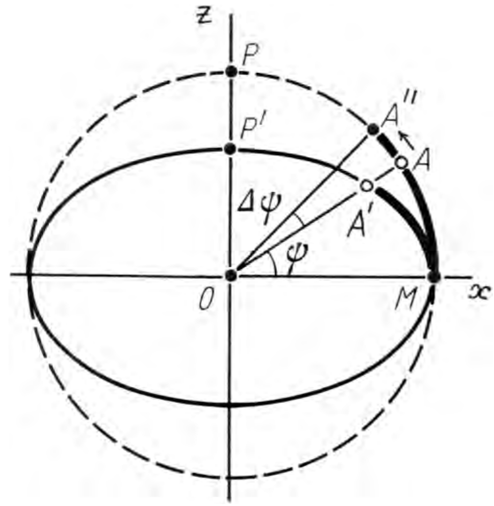

Fig. 7.35. The distorted latitudes of observed stars as a result of minor coordinate system distortion arising from imperfections in the manufacture of the astronomical measuring instruments.

The values of λ_1, λ_2 and λ_3 can be regarded as the semi-axis values of the ellipsoid. In fig. 7.34 these semi-axes are represented with the segments $O\lambda_1$, $O\lambda_2$ and $O\lambda_3$.

8.4. Measurement discrepancies in the "ellipsoidal coordinate system"

Let us proceed to discuss the coordinate changes in the deformed coordinate system as described above – one that we shall be referring to as "ellipsoidal". In fig. 7.35 the plane of the drawing crosses the centre *O*, star *A* and ecliptic pole *P*. This plane intersects the ellipsoid created by the instrument along the ellipse which is drawn in fig. 7.35 as a continuous curve. The respective circumference of an ideal instrument is drawn as a dotted curve. Since we're only interested in the latitudes, let us remind the reader that those are most commonly counted off the ecliptic point, or point *M* in fig. 7.35, used as a point of reference. The observer divided arc *MP'* into 90 equal parts, thus having graded the ring (or ellipse) with degree marks. Since it was an ellipse and not a circle that we graded, the uniform grade marks on the ellipse distort the angles to some extent which therefore makes the grading non-uniform. We are assuming that the observer failed to have noticed this, otherwise the instrument would have been adjusted.

After the observation, the position of the real star *A* was marked by the "elliptic instrument" as *A'*. The observer would consider this to be the real latitude of the star and write it down in his catalogue, which naturally presumes the coordinate system to be ideally spherical; it would therefore become transcribed as a certain point *A"*. The real position of the star would therefore become shifted and lowered somewhat if $1 = \lambda_1 > \lambda_3$.

Should the nature of the ellipse make point *P'* located above point *P* (with $1 = \lambda_1 < \lambda_3$, in other words), the star will be shifted in a different direction. In this case point *A"* shall be higher than point *A* on circumference *PM*. The resulting transformation of the circumference (*A* to *A"*) is naturally of a non-linear nature. It can be continued until the transformation of the entire plane and the entire three-dimensional space. The initial coordinate reference point would remain the same all the time. However, since we consider the distorting effect of the instrument to have been minor, it will suffice to study the linear approximation, as we mention above. In other words, it shall not result in too great an error if we use the main linear part instead of the entire non-linear transformation as described above. This main part is manifest as a stretching by the three orthogonal axes with coefficients of λ_1, λ_2 and λ_3.

b	$\varepsilon =$						
	–0.02	*–0.01*	*–0.004*	*0*	*0.004*	*0.01*	*0.02*
10°	5.0'	3.0'	1.0'	0	–1.0'	–3.0'	–5.0'
20°	11.0'	5.5'	2.0'	0	–2.0'	–5.5'	–11.0'
30°	15.0'	7.5'	3.0'	0	–3.0'	–7.5'	–15.0'
40°	17.0'	8.5'	3.4'	0	–3.4'	–8.5'	–17.0'
50°	17.0'	8.5'	3.4'	0	–3.4'	–8.5'	–17.0'
60°	15.0'	7.5'	3.0'	0	–3.0'	–7.5'	–15.0'

Table 7.4. Quantitatively calculated error values inherent in stellar latitudes and resulting from imperfections in shape of the astrolabe's rings. Here, $\lambda_3 / \lambda_1 = 1 + \varepsilon$. The angular distortion values are given in minutes and fractions of minutes.

We are thus returned to the mathematical formulation of the problem as related above (see sections 8.2 and 8.3). Precise values of the errors introduced into stellar latitudes by this transformation were computed by the authors; the results of the computations are cited in table 7.4.

8.5. Estimating the distortion of angles measured by the "marginally ellipsoidal instrument"

Let us therefore consider a linear transformation of three-dimensional space defined by three values λ_1, λ_2 and λ_3, or the matrix

$$R = \begin{pmatrix} \lambda_1 0 & 0 \\ 0 & \lambda_2 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

We have to estimate the resultant angle distortion. Let ψ equal the true latitude of a real star. If it is measured by an ellipsoidal instrument, it will transform into a different value ψ'. The difference $\Delta\psi = \psi - \psi'$

is the value of the real distortion. Geometrically, the distortion is specified by the angle $\Delta\psi$ between the direction of the real star and the direction measured by a deformed instrument.

We find that it isn't necessary to consider the entire three-dimensional space, and that a flat plane case should suffice after all. Indeed, fig. 7.36 demonstrates that linear transformation R shifts star A into the new position A'', while the parallel of star A shall transform into the parallel of star A''. This is a result of the plane being orthogonal to axis OP and defining the parallel of star A. It will occupy a new position, remaining orthogonal to axis OP. Since it is just the latitudes that we're interested in, it shall suffice to study point B instead of A'' – one that lies upon the meridian of star A, qv in fig. 7.36.

Transformation R makes the plane that crosses axis OP and the meridian of star A rotate around axis OP. The shifted plane generates a linear transformation of the similarity; the three-dimensional problem thus becomes two-dimensional, and so we shall be studying the ellipse in two dimensions, qv in fig. 7.37. Disregarding the previous indications, let us introduce Cartesian coordinates (x, z) to the plane and consider the linear transformation

$$R = \begin{pmatrix} \lambda_1 0 \\ 0 \ \lambda_3 \end{pmatrix},$$

defined by the stretchings λ_1 and λ_3 along the respective axes of x and z.

The position of star A is specified on a unit circumference by radius-vector $a = (x, z)$, and the position of the "shifted star" marked B – by radius-vector $b = (\lambda_1 x, \lambda_3 z)$. Our goal is to calculate the angle $\Delta\psi$ as a function of the initial latitude ψ and stretching (compression) coefficients λ_1 and λ_3.

8.6. Possible distortion estimation and the stability of the resultant dating

According to elementary theorems of analytical geometry, $\cos \Delta\psi$ is equal to the scalar product (a, b) of vectors a and b divided by the length of vector b. The radius of circumference OM is naturally presumed to equal 1, which can always be attained via scale choice. Thus,

$$\cos \Delta\psi = \frac{\lambda_1 x^2 + \lambda_3 z^2}{\sqrt{\lambda_1^2 x^2 + \lambda_3^2 z^2}}.$$

Let $\lambda = \lambda_3 / \lambda_1$ and $\lambda = 1 + \varepsilon$. Then

$$\cos \Delta\psi = \frac{x^2 + \lambda z^2}{\sqrt{x^2 + \lambda^2 z^2}} = \frac{1 + \varepsilon z^2}{\sqrt{1 + 2\varepsilon z^2 + \varepsilon^2 z^2}}.$$

Let $m = 1 / \cos \Delta\psi$, then $m \geq 1$. Squaring shall give us

$$1 + 2\varepsilon z^2 + \varepsilon^2 z^2 = m^2 + 2m^2 \varepsilon z^2 + m^2 \varepsilon^2 z^4.$$

Thus,

$$\varepsilon = \frac{m^2 - 1}{1 - m^2 z^2} + \sqrt{\frac{m^2 - 1}{(1 - m^2 z^2) z^2} + \left(\frac{m^2 - 1}{1 - m^2 z^2}\right)^2}.$$

If the value of $\Delta\psi$ is small, $m \approx 1$ and can be transcribed as

$$m = 1/\cos\Delta\psi \approx 1 + (\Delta\psi)^2/2.$$

Therefore,

$$m - 1 \approx (\Delta\psi)^2/2, \quad 1 - m^2 z^2 \approx 1 - z^2.$$

Finally, for small values of $\Delta\psi$ we have

$$\varepsilon \approx \sqrt{\frac{m^2 - 1}{(1 - m^2 z^2) z^2}} \approx \sqrt{\frac{(m-1)(m+1)}{(1 - z^2) z^2}} \approx \sqrt{\frac{(\Delta\psi)^2}{(1 - z^2) z^2}} = \frac{\Delta\psi}{z\sqrt{1 - z^2}}.$$

However, $z = \sin\psi$ and $\sqrt{1 - z^2} = \cos\psi$, qv in fig. 7.37. And therefore we shall get the following for small values of $\Delta\psi$:

$$\varepsilon \approx \frac{\Delta\psi}{\sin\psi \cos\psi} = \frac{2\Delta\psi}{\sin 2\psi},$$

which implies that $\Delta\psi = \frac{\varepsilon}{2} \cdot \sin 2\psi$.

Now let us find actual numerical estimations of ε. Bear in mind that $\lambda_3 / \lambda_1 = 1 + \varepsilon$, which means the value of ε demonstrates the distortion rate of the coordinate system. The values that we use in our formulae are convenient to express in radians. Thus: $1° = \pi / 180$; $1' = 1° / 60 = 3.14/(60 \times 180) \approx 4.35 \times 10^{-4}$, or $1' \approx 0.00044$.

Therefore, for sensible values of ε, or instrument errors invisible to naked eye, the latitudes of the stars which are close to either the pole or the ecliptic are only marginally distorted. The matter is that $\sin 2\psi$ tends to zero in such cases, which should tell us that sensibly possible instrument errors cannot significantly affect the result of measuring the stars that possess small and large latitudinal values – latitudes close to 0° and 90°, in other words. The maximal latitudinal aberrations are to be expected from the stars located at a large distance from both the pole and the ecliptic pole.

Let us provide the precise quantitative estimates using the actual star catalogue material – the Almagest, for instance. As one can see from fig. 7.27, the maximal latitudinal discrepancy graph of the Almagest's informative kernel grows rather rapidly both on the left and on the right of the interval between 600 A.D. and 1300 A.D. This makes one wonder whether taking the instrument errors into account would allow us to nullify or minimize this latitudinal discrepancy – around the beginning of the new era, for instance, which is the epoch when the Almagest was created, according to the Scaligerian version of chronology.

In other words, we wonder whether one can find any proof to the Scaligerian hypothesis that the Almagest star catalogue was created at some point in time that is close to the beginning of the new era. However, the observer is presumed to have used a somewhat deformed instrument which resulted in a certain error introduced into stellar latitudes. Will taking this error into account permit dating the catalogue to an epoch that will be closer to the beginning of the new era?

We shall demonstrate this to be impossible. Let's presume the measurement results were impaired by the deformed astronomical instruments and take these errors into account in order to minimize the latitudinal discrepancy of the informative kernel of the Almagest under the assumption that the stars were observed around the beginning of the new era. However, we already calculated this discrepancy to be rather substantial – its minimum is 35' for 0 A.D. Can this be rectified by the choice of a fitting ε value?

It was demonstrated above that the minimization of the latitudinal discrepancy for the stars with small and large latitudinal values is hardly possible at all; however, we could try it for the stars whose latitudes are close to 30°-40°. The informative kernel of the Almagest catalogue contains Arcturus; its latitude equals 31 degrees. Furthermore, since Arcturus possesses a high proper movement speed, it is the primary factor to produce the maximal latitudinal discrepancy of the informative kernel around the beginning of the new era. Fig. 7.31 demonstrates that the individual latitudinal discrepancy graph of Arcturus makes this discrepancy reach 35' around the early A.D. period. So let us enquire whether a substantial discrepancy reduction within the vicinity of

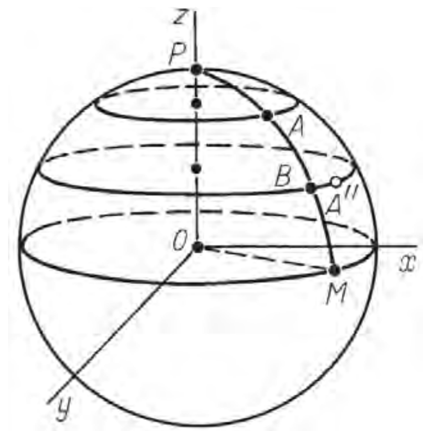

Fig. 7.36. As a result of linear transformation of the coordinate system, the star shall "alter its position" (here $\lambda_1 = 1$).

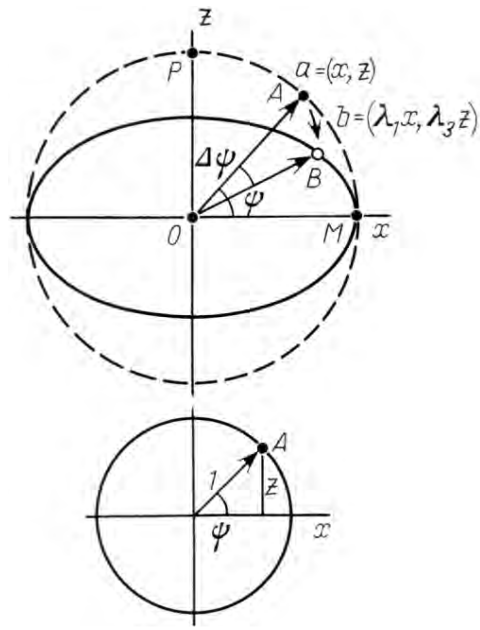

Fig. 7.37. The transformation of a circumference into an ellipsis as a result of a minor coordinate system distortion.

the Scaligerian dating of the Almagest catalogue is a possibility at all, assuming that the observer's instrument was deformed?

Let us calculate the value of ε. As it has been pointed out above, Almagest catalogue precision rate Δ equals 10' as declared by the compiler. Therefore, in order to vanquish the latitudinal discrepancy for Arcturus it has to be reduced from 35' to 10', making the latitude smaller by a factor of 25'. Thus, we have to find such a value of ε that will make $\Delta\psi$ equal 25'. $\Delta\psi = 0.01$ in radians. The formula for ε immediately tells us that

$$\varepsilon \approx \frac{0.01}{\sin 30° \cos 30°} \approx 0.023.$$

Thus, ε should be approximately equal to 0.023. Only such instrumental distortions could explain the latitudinal discrepancy of Arcturus as observed in the early A.D. epoch. However, this value of ε is excessive; for instance, if the radius of an astrolabe equals 50 centimetres, the instrument has to be deformed to such an extent that one of the semi-axes would equal 51 cm; that is to say, the error has to manifest as a 1 cm deformation! One can hardly allow for such low precision of an astronomical device – otherwise we shall also have to assume that cartwheels were made with more precision in Ptolemy's epoch than astrolabes.

8.7. Numerical value table for possible "ellipsoidal distortions"

Above we cite a table of exact distortion values arising from the measurements of stellar latitude made with a certain instrument – an astrolabe, for instance, which would have a deformed latitudinal ring. Let us point out that the latitudinal error rate of star A depends on the value of the real latitude of A as well as the value of $\lambda = R_3 / R_1$. Here R_1 and R_3 are the semi-axes of the instrument's ellipsoidal latitudinal ring. As above, let us assume that $\lambda = 1 + \varepsilon$. Then the value of $\varepsilon = 0$ shall correspond to the ideal ring when the ellipse becomes a circumference. The discrepancies in this case shall equal zero for all the latitudes. As one can see from table 7.4, the maximal absolute values of errors appear at the latitude of 45 degrees, which is also easy to demonstrate theoretically. Table 7.4 contains the values of the difference $b' - b$, where b is the precise value of a star's latitude, and b' – the value of the latitude measured by the marks on the ellipsoidal rings with parameter $\lambda = 1 + \varepsilon$. The values of b and ε are the table entries; the values of distortions $b' - b$ were calculated quantitatively, with the use of a computer.

Table 7.4 demonstrates just what error rate we consider acceptable, replacing the non-linear coordinate grid transformation as considered above by its main linear part. Taking this error into account does not affect our conclusions concerning the impossibility of allowing for Ptolemy's instrument to have been deformed to such an extent that would allow the dating interval to cover the Scaligerian Almagest epoch – I-II century A.D.

8.8. Conclusions

1) It is theoretically possible that a deformed astronomical instrument would produce a spatial coordinate system subject to a certain linear transformation.

2) One can theoretically calculate the dependency between the instrument distortion coefficient ε and the resultant error in stellar latitude estimation.

3) The data contained in actual catalogues (such as the Almagest) allow for an estimation of the numerical values of ε and $\Delta\psi$.

4) No sensible deformations of the astronomical instrument can explain the gigantic latitudinal error discovered in the Almagest catalogue (assuming that the observations were conducted around the beginning of the new era.

9. LONGITUDINAL BEHAVIOUR OF THE NAMED ALMAGEST STARS

We considered the catalogue's latitudes separately from the longitudes in our dating efforts. We discovered that the latitudinal precision of the Almagest is a great deal higher than the longitudinal. It was the analysis of the latitudes that allowed us to build an informative possible dating interval for the Almagest catalogue.

We have naturally conducted all the necessary calculations in order to check the dating that one ends up with using the longitudes instead of the latitudes. As one should have expected if one took the results of our preliminary analysis into account, it turned out that one cannot date the Almagest catalogue to any point on the interval between 1000 A.D. to 1900 A.D. by stellar longitudes, since their precision in the Almagest catalogue is too low.

We shall study the possibility of using both the latitudes and the longitudes for the dating of the Almagest catalogue in the next section.

Let us now regard the dating of the Almagest that we end up with using longitudes and not latitudes as a basis.

We shall use $L_i(t, \gamma, \varphi)$ for referring to the latitude of star i taking into account the rotation angles of the celestial sphere – γ and φ. Bear in mind that what these indications stand for is the compensation of the possible error in the position of the ecliptic. The error is defined by parameters γ and φ. In order to make our conclusions more precise, we shall only consider the 6 named Almagest catalogue stars from celestial area *Zod A* and its immediate vicinity, namely, Arcturus, Regulus, Antares, Spica, Aselli and Procyon. In Chapter 6 we managed to learn that the group error γ coincides with the value of γ_{stat}^{ZodA} for these six stars.

Let us calculate the values of $L_i(t, \gamma_{stat}^{ZodA}(t), \varphi_{stat}^{ZodA}(t))$ for these stars, or their latitudes after the compensation of the respective group error for epoch t. One can naturally make an error here, and a significant one at that for two reasons at the very least. The first is that parameter φ greatly affects the values of the longitudes. At the same time, we have observed that there is no stability in the estimation of this parameter; therefore, one can by no means guarantee that it is the same for all six stars and equals φ_{stat}^{ZodA}. The second reason is as follows. We did not consider group errors in longitude above, which may very well exist, qv in [1339]. Their analysis leads to the necessity of introducing yet another value that would parameterise the group error. Parameter τ can serve as such, qv in Chapter 3. It stands for the celestial sphere's rotation angle around the two new ecliptic poles defined by parameters γ and φ.

Let us define $\Delta L_i(t) = L_i(t, \gamma_{stat}^{ZodA}(t), \varphi_{stat}^{ZodA}(t)) - l_i$. If we draw a function graph for $\Delta L_i(t)$, we could represent it as a sum of an almost linear function (even longitudinal variation resulting from precession) and the irregular "addition" corresponding to all sorts of errors.

Therefore, in order to exclude the effects of precession as well as the possible systematic error τ from consideration, let us introduce the value

$$\Delta\bar{L}(t) = \frac{1}{6}\sum_{i=1}^{6}\Delta L_i(t).$$

$\Delta\bar{L}(t)$ is a rather precise value that measures the longitudinal shifts of the 6 stars under study that result from precession. Let us assume that

$$\Delta L_i^0(t) = \Delta L_i(t) - \Delta\bar{L}(t).$$

The value $\Delta\bar{L}_i^0(t)$ is hardly affected by precession at all.

In fig. 7.38 one sees the changes of $\Delta\bar{L}_i^0(t)$ as functions of the presumed t dating for six Almagest stars considered herein. The first implication of the picture is the low variation velocities of $\Delta\bar{L}_i^0(t)$ values over the course of time. After the compensation of precession, the "fast" stars of the Almagest turn out to be very "slow" insofar as the longitudes are concerned. For instance, the longitude variation velocities of Arcturus and Regulus are almost equal to one another. Procyon becomes the fastest star of six; however, its longitude over 3000 years (between 1100 B.C. and 1900 A.D.) is only altered by 17', which is slightly over 5' per millennium. These slow longitudinal changes are obviously insufficient for an informative dating.

In fig. 7.39 we can see two graphs that could theoretically serve our dating ends. However, the behaviour of these graphs testifies to their utter uselessness in this capacity. Let us consider the two following functions in particular:

$$\Delta L_{\max}(t) = \max_i\left|\Delta L_i^0(t)\right|, \quad \Delta L^0(t) = \max_i \Delta L_i^0(t) - \min_i \Delta L_i^0(t).$$

The first one corresponds to the maximal longitudinal discrepancy between the real stars under study and the ones found in the Almagest. The absolute value of the aberration is considered with the precession accounted for. The second function does not depend on precession, being the difference between the minimal and the maximal aberration. The function of $\Delta L_{max}(t)$ reaches its minimum at $t = 15$, or in 400 A.D.,

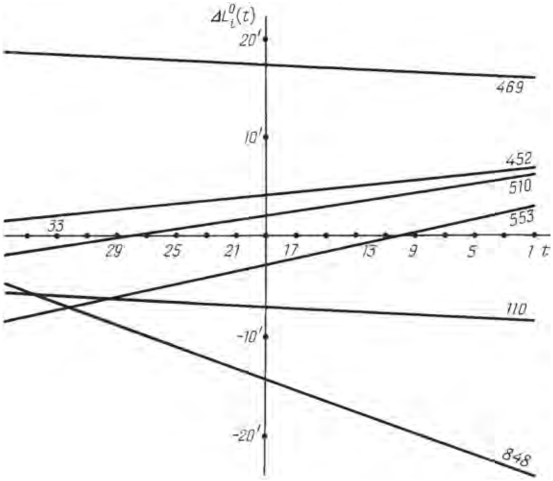

Fig. 7.38. Longitudes of six named stars (Arcturus = 110 in Bailey's enumeration, Regulus = 469, Procyon = 848, Antares = 553, Spica = 510, Aselli = 452) and their behaviour.

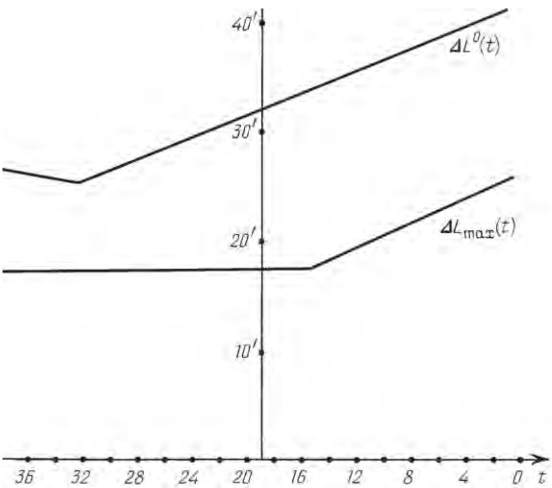

Fig. 7.39. The behaviour of the functions $\Delta L_{max}(t)$ and $\Delta L^0(t)$.

whereas function $\Delta L^0(t)$ does the same at $t = 32.5$, which roughly corresponds to 2350 B.C. Both functions assume considerably large values ($\Delta L^0(t) \geq 25'$, and $\Delta L^0(t) \geq 30'$ starting with the Scaligerian epoch of Hipparchus). Finally, $\Delta L_{max}(t) \geq 17'$. All of this demonstrates latitudinal precision to be too low as compared to proper movement speeds. It doesn't give us any idea as to what the real observation date might be.

Our calculations have thus confirmed that the longitudes of the Almagest catalogue aren't particularly informative due to their low precision rate. The real reason for it was apparently discovered by R. Newton ([614]). He claims that the Almagest longitudes have been forged by someone (also see Chapter 2). We conducted no in-depth research in this direction – it is well possible that a statistical analysis of the longitudes shall detect consecutive patterns in their behaviour. This might demonstrate the existence of group errors in certain parts of the Almagest catalogue, for instance. However, regardless of whether or not this happens to be true, our research demonstrates that it makes no apparent sense to use the longitudes for making the Almagest catalogue dating more precise.

10. THE BEHAVIOUR OF ARC DISCREPANCIES IN THE CONFIGURATION COMPRISED OF THE ALMAGEST INFORMATIVE KERNEL

In Chapter 3 we already mentioned the possibility of dating the catalogue via a comparative analysis of two configurations, one of them immobile and consisting of the Almagest stars, and the other mobile and comprised of modern stars. It was pointed out that this comparison does not require any references to Newcomb's theory – for instance, if it is just the arch distance differences that we have to consider. The use of this method makes us deal with the following hindrances: possible errors in star identification and the low coordinate measurement precision that leads to excessively large dating intervals, as well as the impossibility to differentiate between coordinates measured precisely and imprecisely with such an approach – latitudes and longitudes, for instance.

If we are to choose the Almagest catalogue informative kernel as the configuration under study, the first two hindrances become irrelevant. Indeed, the identity of the stars in question is known for certain, and our primary hypothesis implies their precision to be high enough – latitude-wise, at the very least. Apart from that, the informative kernel contains two stars that move at sufficiently high velocities – Arcturus and Procyon. It is obvious that the unknown error in longitude measurements can lead to dating errors beyond estimation. Nevertheless, the fact that we don't need to consider group errors for this approach makes the corresponding calculations most remarkable. However, it is unfortunately impossible to estimate the errors of these calculations (basing such estimations on our research, at least).

Let us quote the results of the calculations that we have conducted in this direction for 8 of 6 named Almagest stars.

Let l_{ij}^A represent the arc distance between Almagest stars i and j. We shall assume l_{ij}^t to represent a similar distance between modern stars as calculated for observation moment $t = 1, \ldots, 25$. The number of stars in the configuration under study shall be represented by n. Let us mark

$$m_2(t) = \frac{2}{n(n-1)} \sum_{i>j} (l_{ij}^t - l_{ij}^A)^2,$$
$$m(t) = \sqrt{m_2(t)}.$$

The value of $m(t)$ can be considered as the generalized distance between the configuration calculated for epoch t and the respective configuration of the Almagest stars. The minimum points of the functions $m_2(t)$ and $m(t)$ must be close to the catalogue compilation date. In fig. 7.40 one sees the function graphs of $m_2(t)$ and $m(t)$ for a configuration of 8 named stars of the Almagest, and the same graphs for a configuration of 6 named stars in fig. 7.41.

It is obvious that in both cases we see a distinct minimum point that falls upon $t = 14$ (500 A.D.). In both cases the minimal value of $m(t)$ is roughly equivalent to 14', which corresponds to the average precision rate of 10' for every coordinate. The dating of 500 A.D. is very clearly located at a considerable distance from the Scaligerian dating of the Almagest's compilation.

The fact that the dating we end up with, or 500 A.D., is more ancient as compared to the dating interval calculated above with the aid of latitudinal analysis is explained by the fact that the longitudinal error taken independently from the latitudes assumes a minimal value at $t \approx 31$, or 1200 B.C., qv in section 9. Dating the Almagest to 1200 B.C. obviously makes no sense at all. However, one has to bear in mind that the minimum of the average longitudinal discrepancy is manifest very poorly, therefore the precision rate of this dating might equal several millennia. In

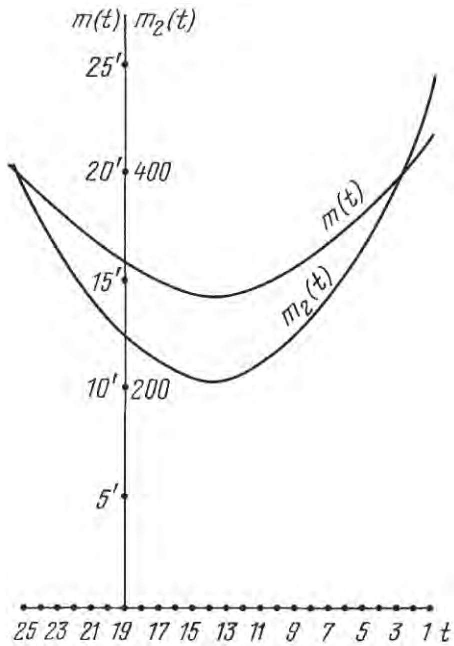

Fig. 7.40. Graphs $m_2(t)$ and $m(t)$ that characterise the varying configuration of 8 named Almagest stars.

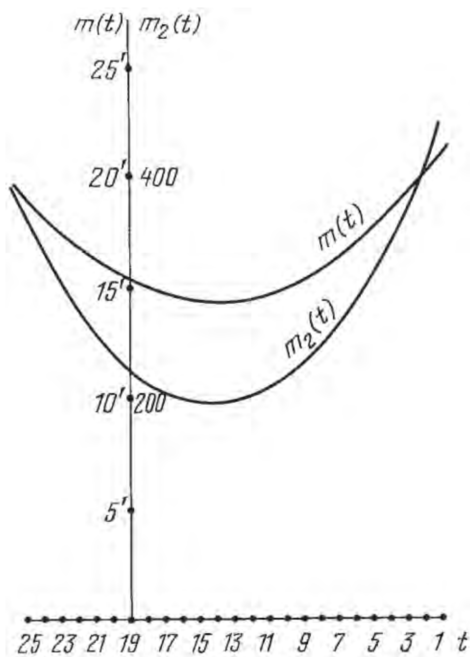

Fig. 7.41. Graphs $m_2(t)$ and $m(t)$ that characterise the altering configuration of 6 named Almagest stars.

other words, it contradicts nothing, qv in figs. 7.38 and 7.39. The minimum of the latitudinal discrepancy, on the other hand, happens to fall on $t = 10$, or 900 A.D., and is a great deal more obvious. This results in the minimum of mean-square arc aberrations falling over the intermediate point $t = 14$, or approximately 500 A.D. This dating is a lot closer to the latitudinal minimum point than to the longitudinal.

11. CONCLUSIONS

1) The dating of the Almagest catalogue estimated with the statistical and the geometrical procedures that we suggest is located on the interval between 600 A.D. and 1300 A.D.

2) A pre-600 A.D. dating gives us no opportunity to make the real celestial sphere concur with the Almagest star atlas, with latitudinal discrepancies of all the stars comprising the informative kernel of the Almagest remaining under the 10" threshold.

3) Even if we are to assume the Almagest catalogue's precision to equal 15' and not 10', the Scaligerian epoch of Ptolemy (I-II century A.D.) remains outside the possible dating interval.

4) Changing the contingent of the Almagest's informative kernel also does not lead to the inclusion of Ptolemy's lifetime in its Scaligerian version into the possible dating interval.

5) Real errors in the manufacture of astronomical instruments leading to non-linear distortions of the celestial sphere in the catalogue can still neither shift nor widen the dating interval enough for the latter to include the Scaligerian epoch of Ptolemy.